Unit - 2 Satellite Orbits

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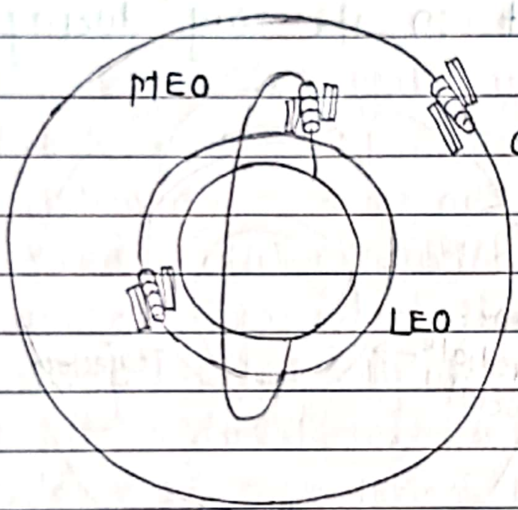
o Introduction :-

- An orbit is the gravitationally curved path of an object around a point in space. While a trajectory is a path traced by a moving body, an orbit is a trajectory that is periodically repeated. While the path followed by the motion of an artificial satellite around Earth is an orbit.



Approx distance

GEO - 35786 km
23 hours 56 min

MEO - 10000-20000 km
6 to 12 hours

LEO - 160 - 500 km
Shorter period

fig :- Example of orbital motion : Satellites revolving around Earth.

Orbiting satellite : Basic principles

- The motion of natural and artificial satellites around Earth is governed by two forces. One of them is the centripetal force directed towards the centre of Earth due to the gravitational force of attraction of Earth, and other is the centrifugal force that acts outwards from the centre of Earth. It may be mentioned here that the centrifugal force is the force exerted during circular motion, by the moving object upon the other object around which it is moving.

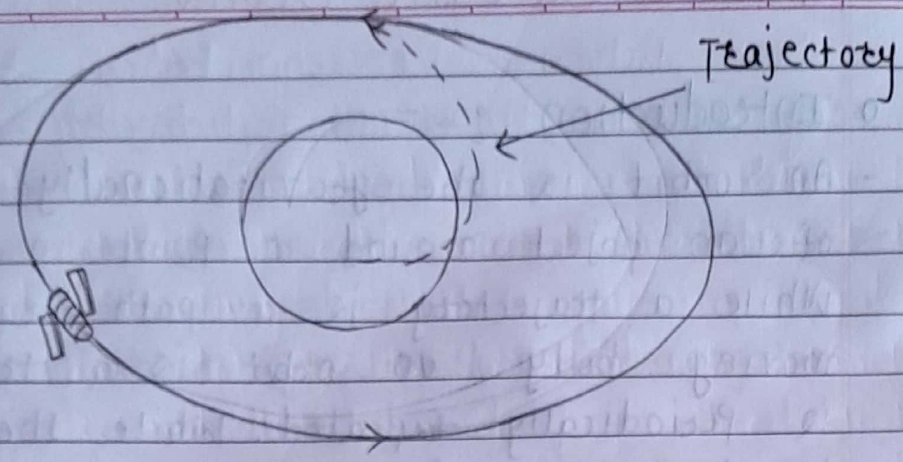


fig :- Example of Trajectory : path followed by a rocket on its way during satellite launch.

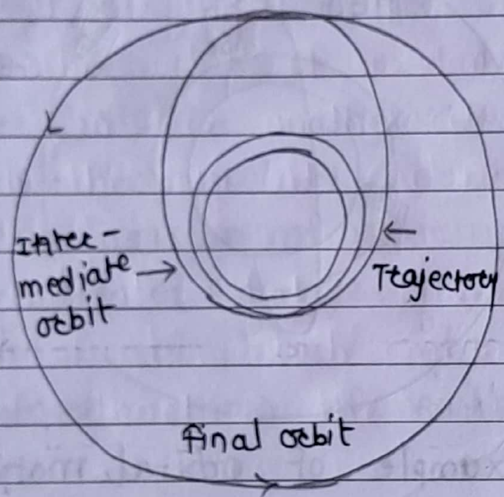


fig :- Example of Trajectory : Motion of a satellite from intermediate orbit to final orbit.

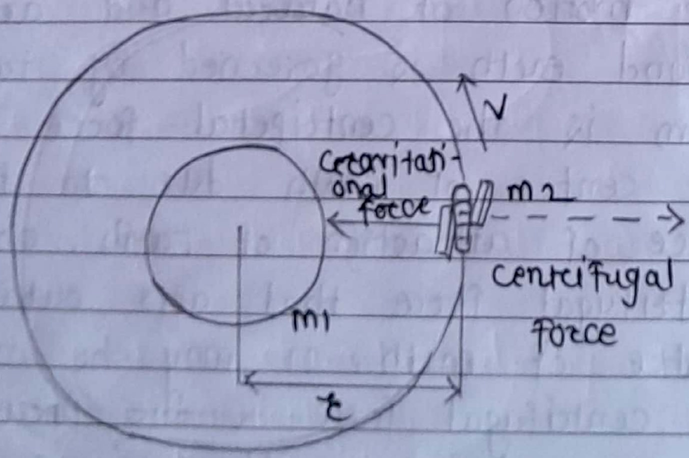


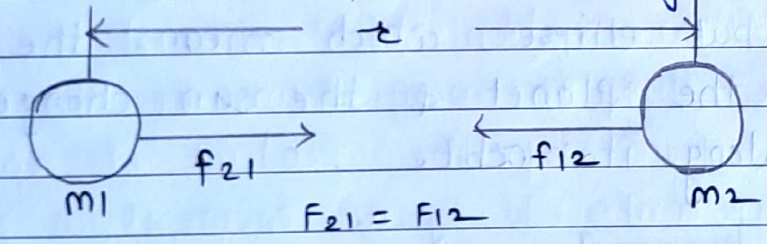
fig :- Gravitational force & the centrifugal force acting on bodies orbiting Earth.

Newton's Law of Gravitation :-

- According to newton's law of ~~gart~~ gravitation, every particle irrespective of its mass attracts every other particle with a gravitational force (F) whose magnitude is directly proportional to the product of the masses of the two particles & inversely proportional to the square of the distance between them,

$$\boxed{F = \frac{G m_1 m_2}{r^2}}$$

Where m_1, m_2 are the masses of particles ;
r is the distance betn two particles ;
G is the gravitational constant
$G = 6.67 \times 10^{-11}\ m^3 / kg s^2$



$F_{21} = F_{12}$

fig :- Newton's Law of Gravitation

Newton's Second Law of Motion :-

- According to Newton's second law of motion, the force equals the product of mass & acceleration.

$$\frac{G m_1 m_2}{r^2} = \frac{m_2 v^2}{r}$$

$$v = \sqrt{\frac{G m_1}{r}} = \sqrt{\frac{\mu}{r}} \quad [\because \mu = G m_1$$
$= 3.986013 \times 10^5\ km^3/s^2$
$= 3.986013 \times 10^{14}\ Nm^2/kg$

- The orbital period $\boxed{T = \frac{2\pi r^{3/2}}{\sqrt{\mu}}}$

[∵ r replace a in elliptical orbit]

Q. State and explain kepler's Three Laws of planetary Motion.

o kepler's Laws :-

- Johannes kepler, based on his lifetime study gave a set of three empirical expressions that explained planetary motion.
- kepler's Laws describe the motion of planets & satellites around the sun or any other celestial body.

kepler's First law :-

- Statement :- The orbit of a planet (or satellite) around the sun (or another celestial body) is an ellipse, with the sun at one of the two foci.

- This law indicates that orbits are not perfect circles but ellipse, which means the distance between the planet & the sun changes as it move along its orbit.

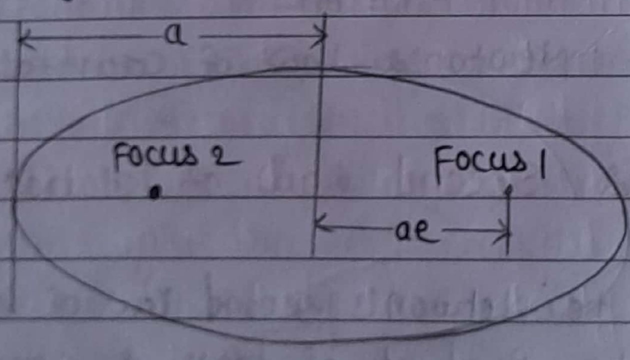


fig :- kepler's first law

- The elliptical orbit is characterized by its semi-major axis a and eccentricity e.
- Eccentricity is ratio of the distance bet^n the centre of the ellipse & either of it foci (= ae)

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- For any elliptical motion, law of conservation of energy is valid at all points on the orbit.
- The law of conservation of energy states that energy can neither be created nor destroyed; it can only be transformed from one form to another.
- In the context of satellites, it means that the sum of kinetic and potential engergies of a satellite always remains constant.

$$= -\frac{Gm_1 m_2}{2a}$$

Where m_1 – Mass of the Earth;
m_2 – Mass of the satellite;
a – semi-major axis of the orbit.

The kinetic & potential energies of a satellite at a distance r from the center of Earth are:-

$$\text{kinetic energy} = \frac{1}{2} m_2 v^2$$

$$\text{Potential energy} = -\frac{Gm_1 m_2}{r}$$

Therefore

$$\frac{1}{2} m_2 v^2 - \frac{Gm_1 m_2}{r} = -\frac{Gm_1 m_2}{2a}$$

$$v^2 = Gm_1 \left[\frac{2}{r} - \frac{1}{a}\right]$$

$$v = \sqrt{\mu \left[\frac{2}{r} - \frac{1}{a}\right]}$$

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Kepler's second law :-

Statement :- The line ~~joing~~ joining the satellite and the center of the Earth sweeps out equal areas in the plane of the orbit in equal times that is, the rate (dA/dt) at which it sweeps area A is constant.

- The rate of change of swept out area is given by

$$\frac{dA}{dt} = \frac{\text{Angular momentum of satellite}}{2m}$$

Where m - mass of satellite.

- Kepler's second law is also equivalent to the law of conservation of momentum, which implies that the angular momentum of the orbiting satellite given by the product of radius vector & the component of linear momentum perpendicular to the radius vector is constant at all point on the orbit.
- The angular momentum of satellite of mass m is given by $= mr^2\omega$

where ω - angular velocity

$$\therefore mr^2\omega = m\omega r r = mv'r$$

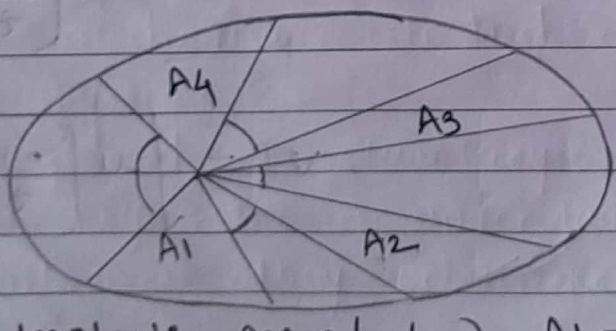

fig :- Kepler's second law $A1 = A2 = A3 = A4$

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- Here v' is the component of satellite velocity v in the direction perpendicular to the radius vector and is expressed as $v \cos\gamma$.
where γ is the angle betⁿ the direction of motion.
Product $rv\cos\gamma$ is constant

$$v_p r_p = v_a r_a = vr\cos\gamma$$

where v_p - velocity at perigee point
r_p - distance at perigee point
v_a - velocity at apogee point
r_a - distance at apogee point
v - velocity at any point in orbit.
r - distance of the point
γ - angle betⁿ the direction of motion.

Kepler's Third Law :-

Statement :- According to kepler's third law, the square of the time period of any satellite is proportional to the cube of the semi-major axis of its elliptical orbit.

Equating the gravitational force with centrifugal force we get

$$\frac{G m_1 m_2}{r^2} = \frac{m_2 v^2}{r} \quad \text{①}$$

Replacing v by ωr in eqⁿ ①

$$\frac{G m_1 m_2}{r^2} = \frac{m_2 \omega^2 r^2}{r} = m_2 \omega^2 r$$

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which gives $\omega^2 = Gm_1/r^3$

substituting $\omega = \frac{2\pi}{T}$, we get

$$T^2 = \left[\frac{4\pi^2}{Gm_1}\right] r^3$$

$$T = \left[\frac{2\pi}{\sqrt{\mu}}\right] r^{3/2}$$

For elliptical orbits provided r replaced by a - semi-major axis.

$$T = \left[\frac{2\pi}{\sqrt{\mu}}\right] a^{3/2}$$

Where, T - orbital period
a - semi-major axis of orbital ellipse
μ - kepler's constant.

Orbital dynamics :-

- Orbital dynamics is the study of motions of artificial satellites and natural celestial bodies under the influence of gravitational forces.
- It is a fundamental aspect of astrodynamics & spaceflight mechanics, combining principles from classical mechanics, celestial mechanics and applied physics.
- In the context of Planetary science, orbital dynam plays a key role in understanding the behaviour of planets as they orbit around the sun.

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1> Kepler's Laws of Planetary Motion :

i> Kepler's First Law (Law of Ellipses) :-

- The orbit of any smaller body about a larger body is always an ellipse, with the center of of mass of the larger body as one of the two foci.

ii> Kepler's Second Law (Law of Equal Areas) :-

- The orbit of the smaller body sweeps out equal areas in equal time.

iii> Kepler's Third Law (Law of Harmonies) :-

- The square of the period of revolution of smaller body about the larger body equals a constant multiplied by the third power of semimajor axis of the orbital ellipse. $T^2 = (4\pi^2 a^3/\mu)$

2> Newton's Law of Motion & Universal Gravitation :-

i> Newton First Law (Inertia) :-

- An object in motion will stay in motion unless acted upon by an external force.

ii> Newton's Second Law ($F = ma$) :-

- The force acting on an object is equal to the mass of the object time its acceleration.

iii> Newton's Third Law (Action and Reaction) :-

- For every action, their is an equal and opposite reaction.

iv> Newton's Law of Universal Gravitation :-

- The Force is proportional to the product of their masses & inversely proportional to square of dist betn them.

$$F = G\frac{m_1 m_2}{r^2}$$

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Q. Write the different orbital characteristics of satellite

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Orbital characteristics :-

- Understanding the characteristics of orbits is essential for satellite operations, space missions, and studying celestial bodies.

1) Orbital shape & size :-

- Semi-major Axis (a) :- The longest radius of an elliptical orbit, representing half the longest diameter. It determines the size of the orbit.
- Eccentricity (e) :- Define the shape of the orbit. values range from 0 (circular orbit) to just under 1 (highly elliptical orbit).

2) Orbital orientation :-

- Inclination (i) :- The tilt of the orbit's plane relative to the equatorial plane of the primary body, measured in degrees. An inclination of 0° means the orbit is equatorial, while 90° indicates a polar orbit.
- Right Ascension of the Ascending Node (Ω) :- The angle from a fixed reference direction to the point where the orbit crosses the equatorial plane going north.
- Argument of periapsis (ω) :- The angle from the ascending node to the point of closest approach to the primary body.

3) Orbital position :-

- True Anomaly (v) :- The angle betn the direction of periapsis & the current position of the orbiting body, measured at the primary focus.

Q. Write short note on satellite spacing and orbital capacity.

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- Mean Anomaly (M) :- An angle that progresses that uniformly with time, used to simplify calculation of the position of a body in its orbit.
- Eccentric Anomaly (E) :- An auxiliary angle used in kepler's equation to relate the mean anomaly to the true anomaly.

4) Orbital Period :-
- The time it takes for a satellite to complete one full orbit around the primary body. It depends on the semi-major axis & the mass of the primary body.

5) Altitude :-
- Perigee :- The closest point in the orbit to the primary body
- Apogee :- The fastest point in the orbit from the primary body.

- Satellite spacing and orbital capacity :-

Satellite Spacing :-
- Satellite spacing refers to the distance betⁿ satellites in the same or adjacent orbits. Proper spacing is critical to avoid collisions & to minimize interference betⁿ satellites especially in heavily utilized orbit.

① Geostationary orbit (GEO) spacing :-
- GEO satellites must maintain precise posⁿ to avoid interference. The ITU regulates orbital slots to ensure adequate spacing.

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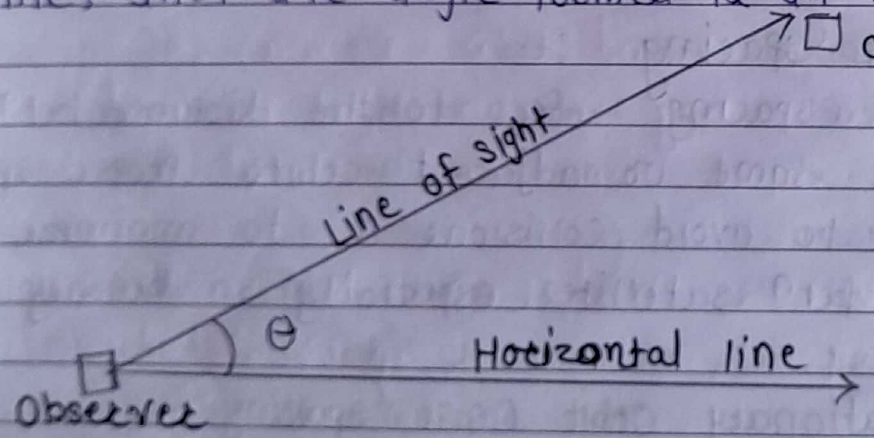
② Low Earth Orbit (LEO) & Medium Earth orbit (ME
spacing :-
- LEO & MEO satellites often operates in large constell'
(eg starlink, oneweb) requiring precise spacing to avoid collisions and ensure coverage.

Orbital capacity :-
- orbital capacity refers to the number of satellites that can be safely & effectively operated within a given orbital region. It depends on several factors orbital altitude, inclination & spacing regulations.
① Physical & Radio Frequency Interference
② Regulatory framework
③ Space debris.
④ Technological advances.

o Angle of elevation :-
- The angle of elevation is an angle that is formed between the horizontal line and the line of sight. If the line of sight is upward from the horizontal line, then the angle formed is an angle of elevation



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Calculating Angle of Elevation :-

- The angle of elevation can be calculated using trignometric funⁿ, typically the tangent funⁿ.

o Height (opposite side) :- The vertical distance from the observer to the object.

o Distance (adjacent side) :- The horizontal distance from the observer to the point directly below the object.

$$\tan(\theta) = \frac{\text{height}}{\text{distance}}$$

Where θ - angle of elevation.

formula $\theta = \tan^{-1}\left(\frac{h}{d}\right)$

$$\theta = \arctan\left(\frac{h}{d}\right)$$

Applications :-

① Surveying :- Measuring the height of buildings, towers & other structure.

② Astronomy :- Determine the posⁿ of celestial objects.

③ Navigation :- Calculating the angle of elevation to determine the direction.

④ Architecture :- Designing structure & ensuring proper sightlines.

o Eclipses :-

- An eclipse occurs when one celestial body moves into the shadow of another celestial body, temporarily blocking its light.

- The major & most frequent source of an eclipse is due to the satellite coming in the shadow of the Earth.

- There are two main types of eclipse observable from Earth : solar eclipse & lunar eclipse.

① Solar eclipse :-

- A solar eclipse happens when the Moon passes bet^{n} the Earth & the sun, casting a shadow on the Earth & partially or completely blocking the sun ligh

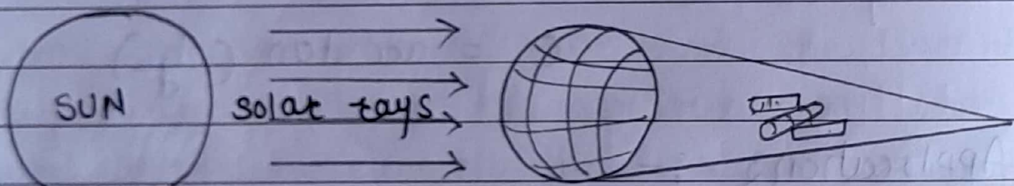

fig :- solar eclipse.

Types of solar eclipse :-

① Total solar Eclipse
② Partial solar Eclipse
③ Angular solar Eclipse
④ Hybrid solar Eclipse.

Q. Explain Launching of satellite in detail.

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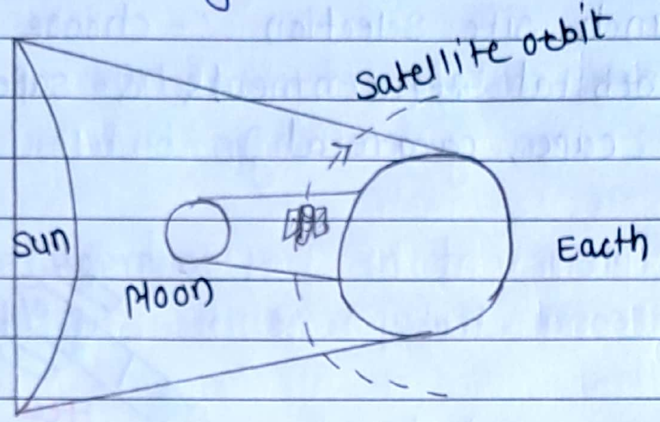
② Lunar Eclipse :-

- A Lunar eclipse occurs when the Earth passes betn the sun & the Moon, casting a shadow on the Moon & causing it to darken.



Types of Lunar Eclipse :-

① Total lunar Eclipse
② Partial Lunar Eclipse
③ Penumbral Lunar Eclipse.

○ Launching and Positioning :-

- Launching and positioning satellites involve multiple stages, from selecting a launch vehicle to precisely placing the satellite in its intended orbit. This process ensures that satellites achieve their mission objectives, whether for communication, navigation, Earth observation.

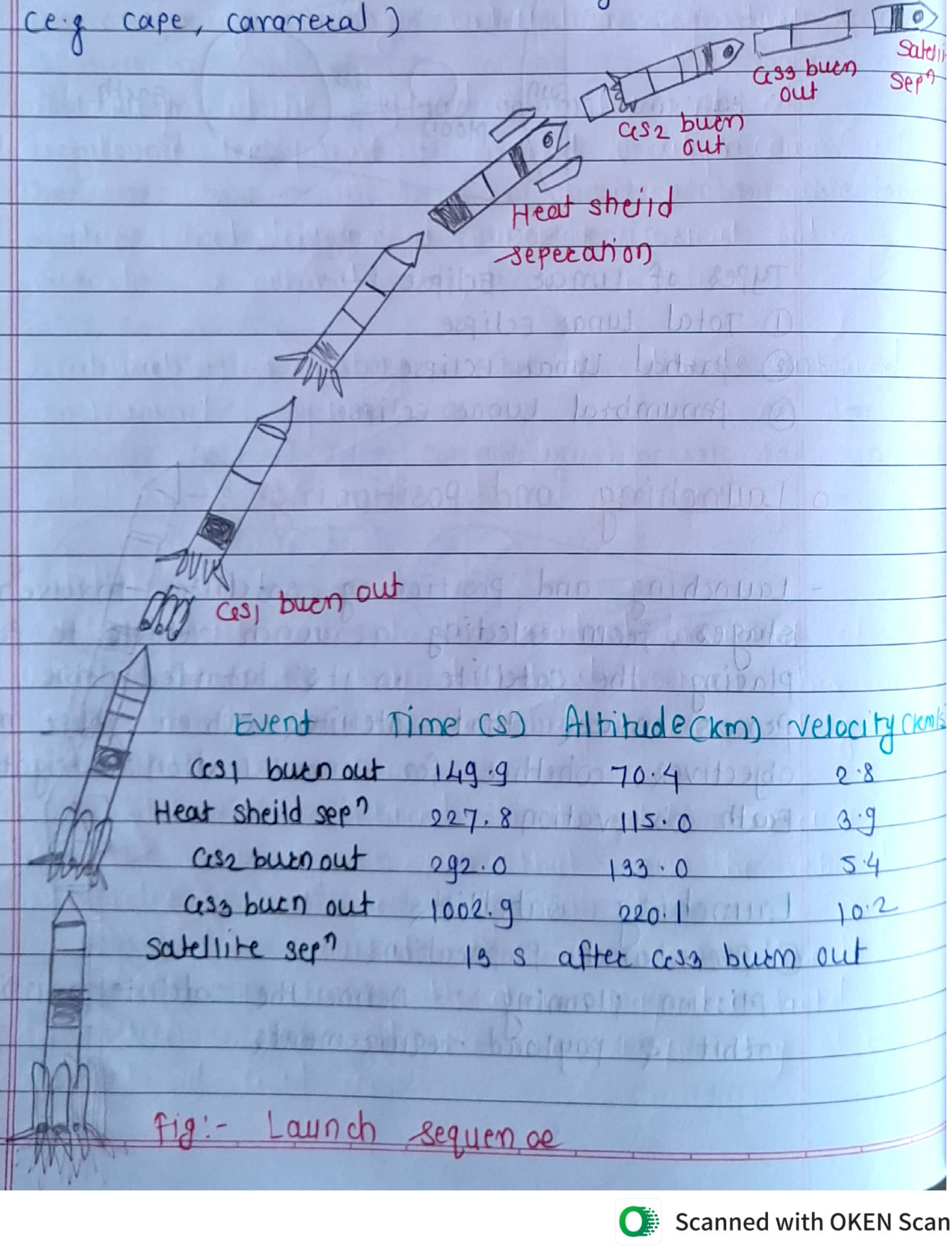
Launching satellites :-

① Pre-Launch Preparations :-

○ Mission planning :- Define the satellite mission, orbit & payload requirements.

o Launch vehicle selection :- choose a rocket based on payload size, weight & target orbit. (e.g. space x Falcon 9, Ariane 5, Atlas V).

o Launch site selection :- choose a launch site based on orbital requirements & safety considerations. (e.g cape, canaveral)



Event	Time (s)	Altitude (km)	Velocity (km/s)
CS1 burn out	149.9	70.4	2.8
Heat sheild sep^n	227.8	115.0	3.9
CS2 burn out	292.0	133.0	5.4
CS3 burn out	1002.9	220.1	10.2
Satellite sep^n	13 s after CS3 burn out		

Fig:- Launch sequence

Q. Explain positioning of the satellite.

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② Launch phases :-

- Countdown & Ignition :- Final checks & fuel loading.

- Ascent : The rocket travel through the atmosphere.

- Orbit Insertion :- The final stage of the rocket places the satellite into a orbit.

③ payload Derolopment :-

- Seperation :- The satellite seperates from the rockets final stage.

- Initial activation :- The satellite deploys solar panned, antennas & othe components & begins transmitting telemetry.

Positioning satellites :-

① Orbit Raising & Circularization :

- Transfer orbit :- Satellite are placed into an elliptical transfer orbit.

- Apogee kick motor :- A rocket motor or propulsion system onboard the satellite fire at the apogee.

② Orbital Adjustments :-

- Inclination changes :- Adjust the tilt of the orbit to match the desired inclination.

- Drift correction :- small adjustments to correct any longitudinal drift.

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③ Station - Keeping :-

o Geostationary Satellite :- Perform regular small maneuvers to maintain position within an assigned orbital slot, countering gravitational perturbations.

o Non - Geostationary satellite :- Adjust orbit to maintain proper phasing withing constellations. (eg starlink, GPS)

④ Deployment of payload & Instruments :-

o Antenna deployment :- Extend & align commⁿ antenna

o Solar panel deployment :- Deploy & orient solar pannels for optimal sunlight exposure.

o Calibration & Testing :- Test & calibrate onboard Instrument & systems.

o Satellite drift and ~~Sat~~ station keeping :-

- Satellite drift :-
- satellite drift refers to the gradual movement of a satellite from its assigned orbital position. This can occur due to various perturbing forces acting on the satellite over time.
- The satellite once placed in its orbit, experiences various perturbing torques that cause the orbital parameter of satellite vary with time.
- These includes gravitational forces from other bodies like solar & Lunar attraction, magnetic field, Interaction, solar radⁿ pressure, assymmetry of earth gravitational field etc.

- Satellite orbiting in low earth orbits also face atmospheric drag & aerodynamic force. Due to this factors the satellite orbit tends to drift & its orientation also changes & hence true orbit of the satellite is different from that defined using Kepler's laws
- The position of satellites east - west location need to be maintained to prevent radio freqⁿ (RF) interference from neighbouring satellites. It may be mentioned here that in the case of geostationary satellite, a 1° drift in east or west dirⁿ.

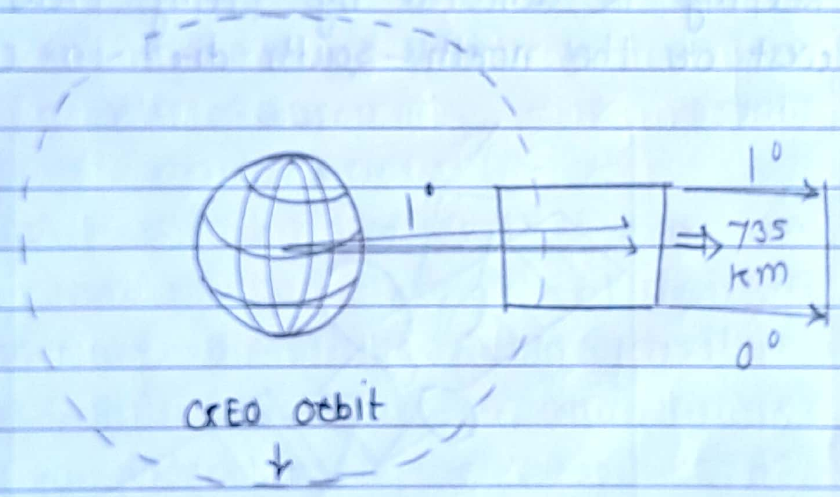

Fig :- Drift of geostationary satellite.

Station keeping :-

- Station keeping is the process of maintenance of satellite's orbit against different factors that cause temporal drift.
- Satellite need to have their orbits adjusted from time to time because the satellite, even though initially placed in the correct orbit, can undergo a progressive drift due to natural forces such as minor gravitational perturbations due to the sun & moon, solar radiation pressure, Earth being an

imperfect sphere, etc.

- The orbital adjustments are usually made by releasing jet of gas or by firing small rockets tied to the body of the satellite.
- In the case of spine - stabilized satellites station keeping in the north - south direction is maintained by firing thrusters parallel to the spin axis in a continuous mode.
- The east - west station keeping is obtained by firing thrusters perpendicular to the spin axis.
- In the case of three axis stabilization, station keeping is achieved by firing thrusters in the east - west or the north - south dirⁿ in continuous mode.

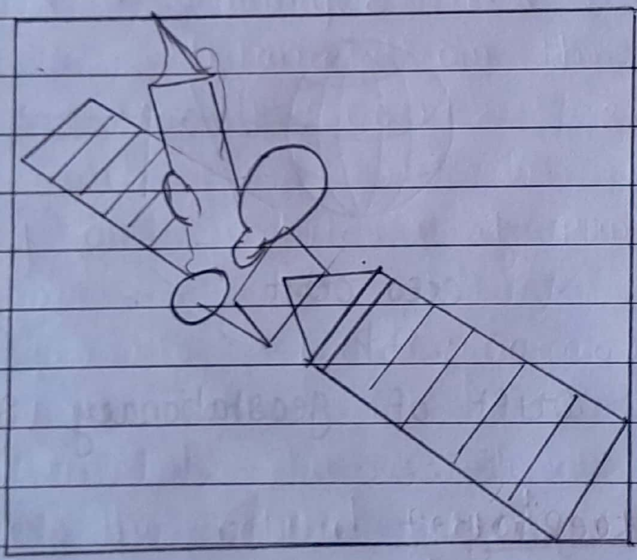

fig :- Three axis stabilized satellite

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Problem :-

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Q.1 A satellite is moving in an elliptical orbit with the major axis equal to 42000 km. If the perigee distance is 8000 km, find the apogee & orbit eccentricity.

⟹ Given :- Major axis = 42000 km

Major axis = Apogee + perigee = 42000 km

∴ Apogee = 42000 − 8000 = 34000 km

Eccentricity is given by

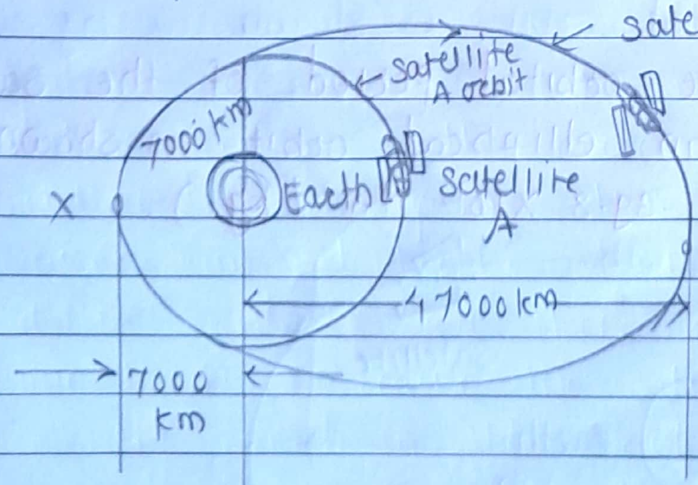
e = (Apogee − Perigee) / Major axis

= (34000 − 8000) / 42000

= 26000 / 42000

e = 0.62

Q.2 Satellite A is orbiting Earth in a circular orbit of radius 7000 km. Satellite B is orbiting Earth in an elliptical orbit with its apogee and perigee distances of 47000 km & 7000 km, respectively. Determines the velocities of the two satellites at point X. (Take $\mu = 39.8 \times 10^{13}\ Nm^2/kg$)



→ velocity of a satellite moving in circular orbit is constant throughout the orbit & is given by

$$v = \sqrt{\mu / R}$$

velocity of satellite A at point x is

$$v = \sqrt{39.8 \times 10^{13} / 7000000} = 7.54 \text{ km/s}$$

velocity of a satellite at any point in an elliptical orbit is given by

$$v = \sqrt{\mu [(2/R) - (1/\alpha)]}$$

Where α - semimajor axis

R - distance from the centre of earth

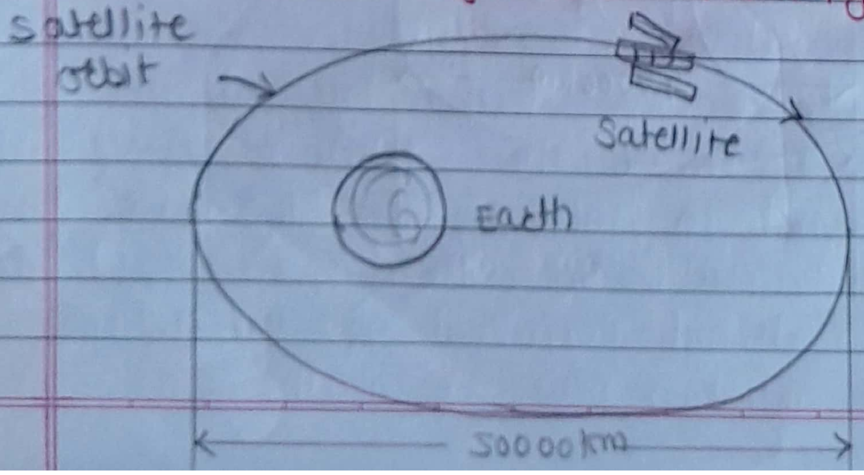
$\alpha = (47000 + 7000)/2 = 27000$ km.

∴ velocity of satellite B at point x is given by

$$v = \sqrt{39.8 \times 10^{13} \times [(2/7000000) - (1 - 27000000)]}$$

$= 9.946$ km/s.

Q 3> Calculate the orbital period of the satellite moving in an elliptical orbit as shown in fig (Take $\mu = 39.8 \times 10^{13}$ Nm²/kg.)



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→ Semi-major axis, $\alpha = 50000/2 = 25000$ km
Orbital time period is given by,

$$T = 2\pi \times \sqrt{\alpha^3/\mu}$$
$$= 2 \times 3.14 \times \sqrt{(25000000)^3/(39.8 \times 10^{13})}$$
$$= 39344 \text{ s} = 10 \text{ h } 55 \text{ min } 44 \text{ s}$$

Q.4) The elliptical orbit of a satellite has its semi-major and semi-minor axes as 25000 km and 18330 km, respectively. Determine the apogee & perigee distances.

→ Given :- semi-major axes = 25000 km
semi-minor axes = 18330 km

$$e = \frac{\sqrt{a^2 - b^2}}{a}$$

$$= \sqrt{[(25000)^2 - (18330)^2]} / 25000$$

$$= 0.68$$

Apogee distance = $\alpha(1+e) = 25000 \times 1.68$
$= 42000$ km

Perigee distance = $\alpha(1-e) = 25000 \times 0.32$
$= 8000$ km.

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Q.1 State & Explain Kepler's three Law's of Planetory motion.

Q.2 Explain concept of orbital dynamics.

Q.3 Write the different orbital characteristics of satellite.

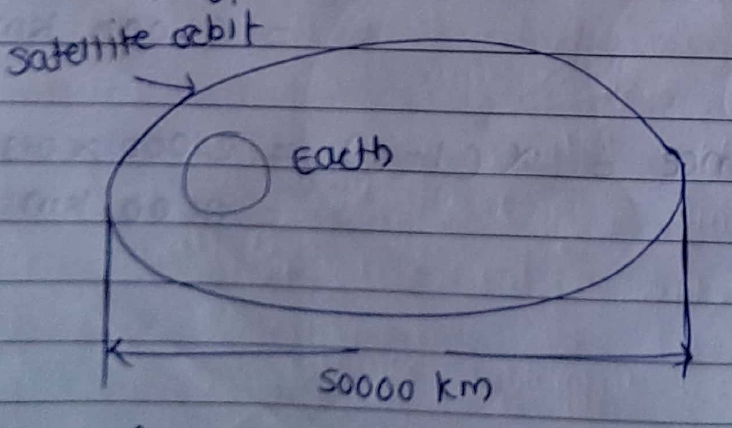
Q.4 Write a short note on satellite spacing & orbital capacity.

Q.5 Explain term :- 1) Angle of Elevation
2) Satellite Drift
3) Station Keeping

Q.6 Explain launching of satellite in detail.

Q.7 Explain positioning of satellite.

Q.8 Calculate the orbital period of the satellite moving in an elliptical orbit as shown in fig. ($\mu = 39.8 \times 10^{13}$ Nm^2/kg)



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