PRACTICE QUESTION PAPER -1

SESSION: 2025-26
CLASS XI
MATHEMATICS (041)

Time Allowed: 3 Hours **Maximum Marks: 80**

General Instructions:

1. This Question Paper contains -five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQs and 02 Assertion -Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 mark each.
4. Section C has 6 Short Answer (SA)-type questions of 3 mark each.
5. Section D has 4 Long Answer (LA)-type questions of 5 mark each.
6. Section E has 3 sources based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

SECTION A

(Multiple Choice Questions- Each question carries 1 mark)

Q1. The sum of 10 items is 12 and the sum of their squares is 18. The standard deviation is

(a) $\frac{1}{5}$ (b) $\frac{2}{5}$

(c) $\frac{3}{5}$ (d) $\frac{4}{5}$

Q2. The value of $1 + i^2 + i^4 + i^6 + \dots + i^{2n}$ is

(a) positive (b) negative

(c) 0 (d) None of the above

Q3. A - (B ∪ C) is

(a) (A - B) ∪ (A - C) **(b) A - B - C**

(c) (A - B) ∩ (A - C) **(d)A - (B ∩ C)'**

Q4. If ${}^nP_5 = 60\,{}^{n-1}P_3$, the value of n is

(a)6 (b)10 (c)12 (d)16

Q5. If $f(x) = x^3 – (1/x^3)$, then $f(x) + f(1/x)$ is equal to

(a) $2x^3$ (b) $2/x^3$ (c) 0 (d) 1

Q6. Name the function, f: R→ R defined as: $\begin{cases} 1 \ if\ x > 0 \\ 0 \ if\ x = 0 \\ -1 \ if\ x < 0 \end{cases}$

(a) Modulus function (b) Signum function

(c) greatest integer function (d) none of these

Q7. The value of sin (45° + θ) – cos (45° – θ) is

(a) 2 cos θ (b) 2 sin θ (c) 1 (d) 0

Q8. What is the real part of $(11 + i)^3$?

a) 1331 b) 1332 c) 1328 d) 1329

Q9. The distance between the lines 3x+4y=9 and 6x+8y=15 is

(a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) $\frac{3}{10}$ (d) $\frac{7}{10}$

Q10. $\lim_{x\to 0} \frac{|\sin x|}{x}$ is equal to

(a) 1 (b) –1 (c) 0 (d) does not exist

Q11. What is a complementary event for "At least one head appears" if two coins are tossed simultaneously?

a) Exactly one head appears b) At least one tail appears

c) At most one tail appears d) None head appears

Q12. The total number of terms in the expansion of $(x+a)^{100}-(x-a)^{100}$after simplification:

(a)50 (b)51 (c)101 (d)100

Q13.**T**he equation of the circle in the first quadrant touching each coordinate axis at a distance of one unit from the origin.

$a)\ x^2+y^2-2x-2y+1=0$ $b)\ x^2+y^2=0$

$c) x^2+y^2+2x-2y+1=0$ $d) x^2+y^2-2x-2y=0$

Q14. the locus of a point for which y = 0, z = 0?

(a)equation of z-axis (b)equation of y-axis

(c)equation of x-axis **(d)All of these**

Q15. The probability of having 53 Sundays in a leap year?

(a)$\frac{3}{7}$ (b)$\frac{5}{7}$ (c)$\frac{7}{7}$ (d)None of these

Q16. The derivative of x^2 cos x is

(a) 2x sin x – x^2 sin x (b) 2x cos x – x^2 sin x

(c) 2x sin x – x^2 cos x (d) cos x – x^2 sin x cos x

Q17. General term of a GP is $9x^{n-1}$ then the common ratio of GP.

(a) 1 (b) x^2 (c) x (d) $\frac{1}{x^2}$

Q18. What is the value of $\lim_{y\to 2} \frac{y^2-4}{y-2}$

(a) 2 (b) 4 (c) 0 (d) cannot be evaluated

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false but R is true.

Q19. Assertion (A) -: Set of English alphabets is the universal set for the set of vowels in English alphabets.

Reason (R): The set of vowels is the subset of consonants in the English alphabets.

Q20. Assertion (A): ${}^{10}C_3 = 120$.

Reason (R): ${}^{n}C_r = \frac{n!}{(n-r)!}$.

SECTION B

(This section comprises of very short answer type-questions (VSA) of 2 marks each)

Q21. The 3rd term of G.P is 4. Then find the product of the first 5 terms.

Q 22. *Evaluate*: $\lim_{x\to 2} \frac{1}{x-2} - \frac{2(2x-3)}{x^3-3x^2+2x}$ **OR** Evaluate : $\lim_{x\to 3} \frac{x^4-81}{2x^2-5x-3}$

Q 23. Find the Centre and the radius of the circle $x^2 + y^2 + 8x + 10y - 8 = 0$

Q 24. Prove that: $\sum_{r=0}^{n} 3^r . {}^nC_r = 4^n$

Q 25. Find the centroid of a triangle, the mid-point of whose sides are D (1,2, – 3), E (3,0, l) and F (-l, 1, -4).

SECTION C

(This section comprises of short answer type questions (SA) of 3 marks each)

Q 26. Let f= $\{\left(x, \frac{x^2}{1+x^2}\right) : x \in R\}$ be a function from R into R. Determine the range of f.

OR

Find the domain of the function $f(x) = \frac{x^2+2x+1}{x^2-8x+12}$

Q 27. Evaluate: $\lim_{x \to \frac{\pi}{2}} \frac{\tan 2x}{x - \frac{\pi}{2}}$

OR

Find $\lim_{x\to 0} f(x)$ and $\lim_{x\to 1} f(x)$, Where $f(x) = \begin{cases} 2x+3 & x \le 0 \\ 3(x+1) & x > 0 \end{cases}$

Q 28. If $(x+iy)^3 = (u+iv)$ then show that If $(x+iy)^3 = (u+iv)$. Than prove that

$\frac{u}{x} + \frac{v}{y} = 4(x^2 - y^2)$

OR

If x+ i y = $\frac{a+ib}{a-ib}$, prove that $x^2 + y^2 = 1$

Q 29. Prove that $\frac{\sin 5x - 2\sin 3x + \sin x}{\cos 5x - \cos x} = \tan x$

OR

$\cot 4x (\sin 5x + \sin 3x) = \cot x (\sin 5x - \sin 3x)$

Q 30. Redefine the function: f(x) = |x – 1| – |x + 6|. Write its domain also.

Q 31. The sum of two numbers is 6 times their geometric mean, show that numbers are in the Ratio (3+2√2): 3-2√2.

SECTION D

(This section comprises of long answer-type questions (LA) of 5 marks each)

Q 32. Mean and standard deviation of 100 observations were found to be 40 and 10, respectively. If at the time of calculation two observations were wrongly taken as 30 and 70 in place of 3 and 27 respectively, find the correct standard deviation.

Q33. Prove that: $\boldsymbol{cos^2} x + \cos^2\left(x + \frac{\pi}{3}\right) + \cos^2\left(x - \frac{\pi}{3}\right) = \frac{3}{2}$

OR

Prove that: $\frac{sec8\theta - 1}{sec4\theta - 1} = \frac{tan8\theta}{tan2\theta}$

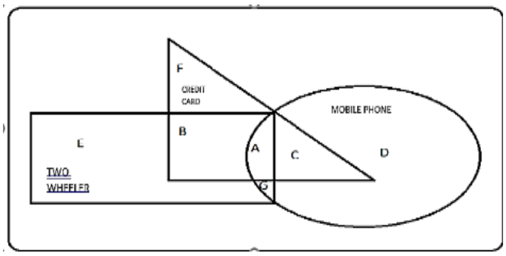
Q34. A solution of 9% acid is to be diluted by adding 3% acid solution to it. The resulting mixture is to be more than 5% but less than 7% acid. If there is 460 litres of the 9% solution, how many litres of 3% solution will have to be added?

Q35. If p and q are the lengths of perpendiculars from the origin to the lines x cos θ – y sin θ = k cos 2θ and x sec θ+ y cosec θ = k, respectively, prove that $p^2 + 4q^2 = k^2$

SECTION- E

Q36. ***Case Study-1***

Of the 200 candidates who were interviewed for a position at a call center, 100 had a two-wheeler, 70 had a credit card and 140 had a mobile phone. 40 of them had both, two-wheeler and a credit card, 30 had both, a credit card and a mobile phone and 60 had both, a two-wheeler and mobile phone and 10 had all three. On the bases of this information Answer the following Questions



Find the no. of Candidates who had Two-wheeler, Credit card and Mobile Phone. 1

(i) What should be the no. of Candidates who had exactly One thing. 1

(ii) How many candidates had none of three. 2

Q37. Two candidates Anil and Surabhi appeared in a written test for a job position in a company. The probability that Anil will qualify the test is 0.05 and that Surabhi will qualify the test is 0.10, The probability that both will qualify the test is 0.02.

Based on the given information, answer the following questions.

(i) Probability that both Anil and Surabhi will not qualify the examination? 1

(ii) Probability that at least one of them will not qualify the examination? 1

(iii) Probability that only one of them will qualify the examination? 2

Q38. A school administration decides to send some of its students of class XI to an educational tour. From a class of 25 students, 10 are to be chosen for the tour. There are three friends - Rajesh, Shreya and Deepa - who decide that either all of them will join or none of them will join. **Based on the above information, answer the following questions.**

(i) In how many ways can the students be chosen for this educational tour, if these three friends will join? 1

(ii) In how many ways can the students be chosen for this educational tour, if these three friends will not join? 1

(iii) Mathematics teacher of school puts some questions for these three students – with a condition that if any one of them answers correctly then, they may join this tour. 2
He asks them to find the number of words formed using all the letters of 'Rajesh'. Deepa answers it correctly. What could be her answer?

OR

Further the teacher asked all of them to find the number of words formed using all letters of 'Deepa'. What could be the correct answer?

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SOLUTION FOR PRACTICE PAPER -1

SESSION: 2025-26

CLASS XI

MATHEMATICS (041)

SECTION -A

Answers: MCQ

Q1. a) Given $\sum x = 12$, $\sum x^2 = 18$ and N=10

Since Standard deviation $\sigma = \frac{1}{N}\sqrt{N\sum x^2 - (\sum x)^2}$

So correct answer is 3/5

Q2. d) Since i^2=-1 and i^3= - i

After applying these values, we have the answer is: None of the above

Q3. c) By using Distributive property for Subtraction on Union.

$A - (B \cup C) = (A - B) \cap (A - C)$

Q4. b) Given that $^nP_5 = 60.{}^{n-1}P_3$

Applying permutation formula $^nP_r = \frac{n!}{(n-r)!}$

Correct Answer is n=10

Q5. c) We have $f(x) = x^3 - (\frac{1}{x^3})$ …. (1)

then $f(1/x) = (\frac{1}{x^3}) - x^3$ ….. (2)

by adding (1) and (2) we find $f(x) + f(\frac{1}{x}) = 0$

Q6. b) Signum function is defined by *f:* R→ R defined as: $\begin{cases} 1 \ if \ x > 0 \\ 0 \ if \ x = 0 \\ -1 \ if \ x < 0 \end{cases}$

Q7. d)To evaluate the value of sin (45° + θ) – cos (45° – θ) we apply formulas
Sin (A+B) = SinA.CosB +CosA .SinB
And Cos (A-B) = CosA.CosB+SinA.SinB
By putting $A=45^0$ and $B=\theta$
We find the correct Answer 0

Q8. c) Explanation: $(11 + i)^3$ = 113 + 3.112.i +3.i2.11 +i3

= 1331 + 363i – 3 – i

= 1328 + 365i.

Q9. c) The distance between the lines 3x+4y=9 and 6x+8y=15

Applying perpendicular distance formula from origin to a line $d = \frac{c}{\sqrt{a^2+b^2}}$

$d_1 = \frac{9}{\sqrt{(-3)^2+(-4)^2}} = \frac{9}{5}$ and $d_2 = \frac{15}{\sqrt{(-6)^2+(-8)^2}} = \frac{15}{10} = \frac{3}{2}$

so distance between both lines is $d=d_1-d_2=\frac{9}{5}-\frac{3}{2} = \frac{3}{10}$

Q10. d) Since $|sin\ x| = \pm sin\ x$

So, for it Right Hand Limit ≠ Left Hand Limit

Therefore, its limit Does not exist

Q11. d) By the definition of complementary event the correct Answer for "At least one head appears" if two coins are tossed simultaneously is None head appears

Q12. a) No. of terms in the expansion of $(x+a)^{100} - (x-a)^{100}$

Applying Binomial Theorem $(x+a)^n - (x-a)^n = 2[{}^nC_1.x^{(n-1)}.a^1 + {}^nC_3.x^{(n-3)}.a^3 + {}^nC_5.x^{(n-5)}.a^5 + \ldots\ldots]$

Here total no. of terms in this expansion =50

Q13. a) The equation of the circle in the first quadrant touching each coordinate axis at a distance of one unit from the origin is given by $(x-1)^2 + (y-1)^2 = 1^2$

So we find $x^2 + y^2 - 2x - 2y + 1 = 0$

Q14 .c) For a Point located on x axis, y=0=z

Hence its locus is x-axis

Q15. a) There are 366 days in leap year means 52 weeks and 2 extra days. Make the possibilities for two extra days

and evaluate the probability. Two extra days will be:

{Monday, Tuesday}, {Tuesday, Wednesday}, {Wednesday, Thursday}, {Thursday, Friday}, {Friday, Saturday}, {Saturday, Sunday} and {Sunday, Monday}

In two of the cases {Saturday, Sunday} and {Sunday, Monday}, Sunday is present, therefore favourable

outcomes will be 2 and total possibilities are 7, therefore, total outcomes will be 7.

$P(A)=\frac{2}{7}$

Q16. b) let $h(x)= x^2 \cos x$

applying product formula for derivative,

$f(x)= x^2$ and $g(x)=\cos x$

$h(x)=f(x).g(x)$ then

$h'(x) = f(x).g'(x) + f'(x).g(x)$

$h'(x)=2x \cos x - x^2 \sin x$

Q17. c) Given that General term of a GP is $9x^{n-1}$

Then $T_1= 9x^{(1-1)}=9$

$T_2= 9x^{(2-1)}=9x$

therefore common ratio of GP=T_2/T_1.=x

Q18. b) The value of $\lim_{y\to 2} \frac{y^2-4}{y-2}$

$= \lim_{y\to 2} y-2.y+2y-2 = \lim_{y\to 2}(y+2) = 4$

ASSERTION-REASON BASED QUESTIONS

Q19.c) A is true but R is false.

Q20.a) Both A and R are true and R is the correct explanation of A.

SECTION-B

Q21. Solution:

Third term is given as 4. So, we have:

$T_3=4$

⇒ $ar^{3-1}=4$

⇒ $ar^2=4$

The product of first five terms is: $a.ar.ar^2 \cdot ar^3 \cdot ar^4=a^5r^{10}=(ar^2)^5$

Purring the value of $ar^2=4$

In above equation ,we will get product $=(4)^5=1024$

Q22. Solution:

$$\lim_{x\to 2}\frac{1}{x-2}-\frac{2(2x-3)}{x(x-1)(x-2)}$$

$$=\lim_{x\to 2}\frac{x^2-5x+6}{x(x-1)(x-2)}$$

$$=\lim_{x\to 2}\frac{(x-2)(x-3)}{x(x-1)(x-2)}=\lim_{x\to 2}\frac{(x-3)}{x(x-1)}=-\frac{1}{2}$$

OR

$$=\lim_{x\to 3}\frac{x^4-81}{2x^2-5x-3}$$

$$=\lim_{x\to 3}\frac{(x^2+9)(x+3)(x-3)}{(x-3)(2x+1)}=\lim_{x\to 3}\frac{(x^2+9)(x+3)}{(2x+1)}=\frac{108}{7}$$

Q23. Solution:

The given equation is $x^2+y^2+8x+10y=8$

Now completing squares, we get

$(X^2+8x+16)+(y^2+10y+25)=8+16+25=49$

$(x+4)^2+(y+5)^2=7^2$

Therefor the geven circle has centre at (-4, -5) and radius 7.

Q24. Solution:

$\sum_{r=0}^{n}3^r n_{c_r}=1+3\,n_{c_1}+3^2\,n_{c_2}+3^3 n_{c_3}+\ldots\ldots\ldots\ldots\ldots+3^n$

Now we have by Binomial expansion

$4^n=(1+3)^n=1+3\,n_{c_1}+32\,n_{c_2}+33n_{c_3}+\cdots\ldots\ldots\ldots\ldots+3^n$

Q25. Solution:

For a triangle the coordinates of the centroid is also given by the average of the coordinates of midpoints of its sides. If A (a, b, c), B (d, e, f) and C (g, h, l) are midpoints of its sides of any triangle then the coordinates of the centroid are given by $x=\frac{a+d+g}{3}, y=\frac{b+e+h}{3}, z=\frac{c+f+l}{3}$

By applying it we will have (1, 1, −2) is the required centroid

SECTION C

Q26. Solution:

$$here\ \ y=f(x)=\frac{x^2}{1+x^2}$$

$$x=\pm\sqrt{\frac{y}{1-y}}$$

$$but\ \frac{y}{1-y}>0$$

$y \in [0,1)$

OR

f(x) is a rational function of x

f(x) assumes real values of all x except for those values of x for which

x2 - 8x + 12 = 0

(x – 6) (x – 2) = 0 so domain of function = R – {2, 6}

Q27. Solution:

Put $x - \frac{\pi}{2} = y$ with changing the limit for y then we have $\lim_{y \to 0} \frac{\tan(\pi+2y)}{y} = \lim_{y \to 0} 2\frac{\tan 2y}{2y} = 2$

the correct limit 2

OR

Find $\lim_{x \to 0} f(x)$ and $\lim_{x \to 1} f(x)$,

Where $f(x) = \begin{cases} 2x+3 & x \le 0 \\ 3(x+1) & x > 0 \end{cases}$

For the limit of f(x) at x=0

$\lim_{x \to 0} f(x) = \lim_{x \to 0} (2x+3) = 3$

Now to finding correct limit for f(x) at x=1

$\lim_{x \to 1} f(x) = \lim_{x \to 0} 3(x+1) = 3(1+1) = 6$

So correct limit for f(x) at x=1 is 6

Q28. Solution:

We have

$(x+iy)^3 = (u+iv)$ and we have to prove that

$\frac{u}{x} + \frac{v}{y} = 4(x^2 - y^2)$

$x^3 - 3xy^2 + i(3x^2y - y^3) = (u+iv)$

$x^3 - 3xy^2 = u$

$3x^2y - y^3 = v$

Hence $\frac{u}{x} + \frac{v}{y} = 4(x^2 - y^2)$

OR

We have x+ i y = $\frac{a+ib}{a-ib}$,and to prove that $x^2 + y^2 = 1$

Now x+ i y = $\frac{a+ib}{a-ib}$ -(1)

Then x- i.y = $\frac{a-ib}{a+ib}$ --(2) ,

then by product of (1) and (2) (x+ iy) (x- iy) = $\frac{a+ib}{a-ib} \cdot \frac{a-ib}{a+ib}$

$x^2 + y^2 = \frac{a^2+b^2}{a^2+b^2} = 1$

Q29. Solution:

To prove that $\frac{\sin 5x - 2\sin 3x + \sin x}{\cos 5x - \cos x} = \tan x$

$$\frac{(\sin 5x + \sin x) - 2\sin 3x}{(\cos 5x - \cos x)} = \frac{2\sin 3x . \cos 2x - 2\sin 3x}{-2\sin 3x . \sin x}$$

$$\frac{2\sin 3x.(\cos 2x - 1)}{-2\sin 3x . \sin 2x} = \frac{(\cos 2x - 1)}{-\sin 2x} = \frac{1 - 2\sin^2 x - 1}{-2\sin x \cos x} = \tan x$$

OR

To prove cot 4x (sin 5x + sin 3x) = cot x (sin 5x – sin 3x)

We can write it as:

$$\frac{sin5x + sin3x}{sin5x - sin3x} = \frac{cot4x}{cotx}$$

Now $\frac{sin5x+sin3x}{sin5x-sin3x} = \frac{2sin4x.cosx}{2cos4x.sinx}$

$= \frac{cot4x}{cotx}$

By cross multiplication,

We have cot 4x (sin 5x + sin 3x) = cot x (sin 5x – sin 3x)

Q30. Solution:

Given function is f(x) = |x – 1| – |x + 6|

Redefine of the function is:

$$f(x) = \begin{cases} -x + 1 + x + 6, & x \leq -6 \\ -x + 1 - x - 6, & -6 \leq x < 1 \\ x - 1 - x - 6, & x \geq 1 \end{cases}$$

$$= \begin{cases} 7, & x \leq -6 \\ -2x - 5, & -6 \leq x < 1 \\ -7, & x \geq 1 \end{cases}$$

The domain of this function is R.

Q31. Solution:

Let the two numbers be a and b.

Then its G.M. = √ ab

According to the given condition,

⇒ a + b = 6√ab .... (1)

Finding a - b= 4√ 2 √ ab ... (2)

from (1) and (2), we obtain

a = (3 + 2√2) √ ab

Substituting the value of a in (1),

we obtain b = (3 - 2√2) √ ab

Hence the ratio of the numbers is

$\frac{a}{b} = \frac{[(3 + 2\sqrt{2}).\sqrt{ab}]}{[(3 - 2\sqrt{2})\sqrt{ab}]} = \frac{(3 + 2\sqrt{2})}{(3 - 2\sqrt{2})}$

Thus, the required ratio is (3 + 2√2): (3 - 2√2)

SECTION D

(Long answer type questions (LA) of 5 marks each)

Q32. Solution:

We have the following information:

- Mean (x̄) = 40
- Standard Deviation (σ) = 10
- Number of observations (n) = 100
- Wrong observations taken: 30 and 70
- Correct observations: 3 and 27

The mean is given by the formula: Mean=Σxn

From this, we can calculate the total sum of observations (Σx):

Σx=Mean×n=40×100=4000

The incorrect observations were 30 and 70, while the correct observations should have been 3 and 27. We need to adjust the total sum:

Correct Sum=Σx−30−70+3+27
Calculating this gives:
Correct Sum=4000−100+30=3930
Now, we can calculate the correct mean using the adjusted sum:
Correct Mean=3930100=39.30
We know that: σ2=Σx2n−(Σxn)^2
Given that the original standard deviation is 10, we can find Σx²:
102=Σx^{2}100−402
This simplifies to: 100=Σx^{2}100−1600
Thus, Σx^2=1700×100=170000
Now we need to adjust Σx² for the incorrect entries:
Correct Σx2=170000−302−702+32+272
Calculating the squares:
302=900,702=4900,32=9,272=729
Now substituting these values:
Correct Σx2=170000−900−4900+9+729
Calculating this gives:
Correct Σx2=170000−900−4900+738=164938
Now we can find the correct standard deviation using the adjusted values:
σ2=164938100−(3930100)^2
Calculating each term: σ2=1649.38−1544.49
Thus, σ2=104.89
Taking the square root gives:
σ=√104.89≈10.241

Q33. Solution:

$$\text{L.H.S.} = \frac{1+\cos 2x}{2} + \frac{1+\cos\left(2x+\frac{2\pi}{3}\right)}{2} + \frac{1+\cos\left(2x-\frac{2\pi}{3}\right)}{2}$$

$$= \frac{1}{2}\left[3 + cos2x + 2cos2x.\cos\left(\pi - \frac{\pi}{3}\right)\right]$$

$$= \frac{1}{2}[3 + cos2x - cos2x] = \frac{3}{2}$$

OR

$$\frac{\sec 8\theta - 1}{\sec 4\theta - 1} = \frac{\frac{1}{\cos 8\theta} - 1}{\frac{1}{cos4\theta} - 1}$$

$$= \frac{(1-cos8\theta)cos4\theta}{cos8\theta(1-\cos 4\theta)} = \frac{2sin^2 4\theta \cos 4\theta}{\cos 8\theta(1-\cos 4\theta)}$$

$$= \frac{2\sin 4\theta \cos 4\theta \sin 4\theta}{cos8\theta(1-\cos 4\theta)} = \frac{\tan 8\theta \sin 4\theta}{1-\cos 4\theta}$$

$$= \frac{\tan 8\theta \; 2\sin 2\theta \cos 2\theta}{2sin^2 2\theta} = \tan 8\theta \cot 2\theta = \frac{\tan 8\theta}{\tan 2\theta}$$

Q34. Solution:
Let x be the amount of 3% acid solution to be added (in liters)
- The amount of acid in the 460 L of 9% solution:
Acid from 9% solution=0.09×460=41.4 L
- The amount of acid in x L of 3% solution:
Acid from 3% solution=0.03×x=0.03x L
The total volume of the mixture after adding x liters of 3% solution is: 460+x L

The total amount of acid in the mixture is:
41.4+0.03x L
We want the concentration of acid in the resulting mixture to be more than 5% and less than 7%. This gives us two inequalities:
1. For more than 5%: $41.4+0.03x460+x>0.05$
2. For less than 7%: $41.4+0.03x460+x<0.07$
Multiply both sides of the first inequality by 460+x (assuming $460+x>0$):
$41.4+0.03x>0.05(460+x)$
Expanding the right side:
$41.4+0.03x>23+0.05x \Rightarrow x<920$
Now, solve the second inequality: $41.4+0.03x<0.07(460+x)$
Expanding the right side: $41.4+0.03x<32.2+0.07x \Rightarrow 9.2<0.04x$
Dividing by 0.04: $x>230$

Combine the Results

From the two inequalities, we have:
$230<x<920$

Q35. Solution:

Equations of given lines are: $x\cos\theta - y\sin\theta = k, \cos 2\theta$ … (1)
and $x\sec\theta - y\,\mathrm{cosec}\theta = k$ … (2)
The perpendicular distance of a point (x_1, y_1) from the line ax +by +c =0 is given by $\left|\frac{ax1+by1+c}{\sqrt{a^2+b^2}}\right|$
Applying this formula for line (1) and (2) we have
$p = k.\cos 2\theta$ .... (3)
$q = k.\sin 2\theta$ .... (4)
Now $p^2 + 4q^2 = k^2\cos^2 2\theta + k^2\sin^2 2\theta$
$p^2 + 4q^2 = k^2(\cos^2 2\theta + \sin^2 2\theta)$
$p^2 + 4q^2 = k^2$

SECTION-E

Q36. Solution:

(i) The no. of Candidates who had Two-wheeler, Credit card and Mobile Phone (A)=10
(ii) The no. of Candidates who had exactly One thing (E+F+D) =80
(iii) Total number of candidates = 200.
Number of candidates who had at least one of the three = n (A ∪ B ∪ C), where A is the set of those who have a two-wheeler, B is the set of those who have a credit card, and C is the set of those who have a mobile phone.

Therefore, n (A ∪ B ∪ C) = 100 + 70 + 140 - {40 + 30 + 60} + 10
Or n (A ∪ B ∪ C) = 190.
n (A ∪ B ∪ C) is the number of candidates who had at least one of three.
As 190 candidates who attended the interview had at least one of the three, (200 - 190 = 10) candidates had none of three.

Q37. Solution: -

(i) $P(AUB) = P(A) + P(B) - P(A \cap B)$
$P(AUB) = 0.05 + 0.10 - 0.02 = 0.13$
Since $P(AUB) + P(\overline{AUB}) = 1$

$P(\bar{A} \cap \bar{B}) = 1 - P(AUB) = 1 - 0.13 = 0.87$

(ii) $P(AUB) = P(A) + P(B) - P(A \cap B)$

$P(AUB) = 0.05 + 0.10 - 0.02 = 0.13$

(ii) $P(A \cap \overline{B}) + P(\overline{A} \cap B) = 0.03 + 0.08 = 0.11$

Q38. Solution:

(i)In how many ways can the students be chosen for this educational tour, if these three friends will join?

Ans: Number of ways can the students be chosen for this educational tour, if these three friends will join $^{22}C_7$

(ii)In how many ways can the students be chosen for this educational tour, if these three friends will not join?

Ans: No. of ways can the students be chosen for this educational tour, if these three friends will not join $^{22}C_{10}$

(iii)The Mathematics teacher of school puts some questions for

these three students - with a condition that if any one of them answers correctly then, they may join this tour.

He asks them to find the number of words formed using all the letters of 'Rajesh'. Deepa answers it correctly. What could be her answer?

Ans: The number of words formed using all the letters of 'Rajesh'.**720**

OR

Further the teacher asked all of them to find the number of words formed using all letters of 'Deepa'. What could be the correct answer?

Ans: The number of words formed using all letters of 'Deepa'**60**

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PRACTICE QUESTION PAPER-2

SUBJECT: MATHEMATICS (041)

TIME: 3 HOURS **CLASS - XI** **Max. Marks. -80**

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA) type questions of 2 marks each.
4. Section C has 6 Short Answer (SA) type questions of 3 marks each.
5. Section D has 4 Long Answer (LA) type questions of 5 marks each.
6. Section E has 3 Source Based/Case Based/Passage Based questions of 4 marks each with sub-parts.

SECTION – A (1×20=20)

1. Which of the following collection of objects is not a set?
(a) The collection of all even integers.
(b) The collection of all months of a year beginning with letter J.
(c) The collection of most talented writers of India.
(d) The collection of all prime numbers less than 20.

2. On real axis if A = [1, 5] and B = [3, 9] , then A – B is
(a) (5, 9) (b) (1, 3)
(c) [5, 9) (d) [1, 3)

3. Which of the following statement is false?
(a) $A - B = A \cap B'$ (c) $A - B = A - B'$
(b) $A - B = A - (A \cap B)$ (d) $A - B = (A \cup B) - B$

4. Let n(A) = m and n(B) = n, then the number of non-empty relations from A to B is
(a) m^n (c) $2^{mn} - 1$
(b) n^m-1 (d) 2^{mn}

5. Which of the following ralations is a function?
(a) R = {(4,6), (3,9), (-11, 6), (3, 11)} (c) R = {(2,1), (4,3), (6, 5), (8, 7), (10,9)}
(b) R = {(1,2), (2,4), (2, 6), (3, 5)} (d) R = {(0,1), (1,3), (2, 4), (3, 1), (3,5)}

6. The domain and range of the real function f defined by f(x) = $\sqrt{x-1}$ are
(a) Domain= (1, ∞), Range = (0, ∞) (c) Domain= [1, ∞), Range = [0, ∞)
(b) Domain= [1, ∞), Range = (0, ∞) (d) Domain= (1, ∞), Range = [0, ∞)

7. A wheel makes 450 revolutions per hour. The number of radians through which it turns in one second is
(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{3}$

8. The value of tan 1°.tan 2°.tan 3°…tan 89° is
(a) 0 (b) 1 (c) 2 (d) $\frac{1}{2}$

9. Which of the following is not correct?
(a) sin θ = -$\frac{1}{5}$ (c) sec θ = $\frac{1}{2}$
(b) cos θ = 1 (d) tan θ = 20

10. If n is any integer, then the value of $\frac{i^{4n+1} - i^{4n-1}}{2}$ is
(a) i (b) 0 (c) 1 (d) $-i$

11. If $10 \le -5(x-2) < 20$, then x belongs to
(a) (-2, 0] (b) (-2, 0)

(c) [-2, 0) (d) [-2, 0]

12. The length of the rectangle is double the breadth. If the minimum perimeter of the rectangle is 120 cm, then

(a) breadth $> 20\ cm$ (c) breadth $\leq 20\ cm$

(b) breadth $< 20\ cm$ (d) breadth $\geq 20\ cm$

13. The number of ways in which 5 boys and 3 girls can be seated in a row, so that no two girls sit together is

(a) 8! (c) $3! \times {}^5P_4$

(b) $5! \times 3!$ (d) $5! \times {}^6P_3$

14. The value of $9^{1/3} \times 9^{1/9} \times 9^{1/27} \times$ …………….to ∞ is

(a) 1 (b) 3 (c) 9 (d) none of these

15. The distance of the point P(3,4,5) from yz-plane is

(a) 3 units (c) 5 units

(b) 4 units (d) none of these

16. $\lim_{x \to \frac{3}{2}} [x]$ is equal to

(a) 1 (b) – 1 (c) 2 (d) does not exist

17. $\lim_{x \to 2} \frac{\log(x-1)}{x-2}$ is equal to

(a) 0 (b) -1 (c) $\frac{1}{2}$ (d) 1

18. A coin is tossed twice, the probability of getting at least one tail is

(a) $\frac{1}{4}$ (b) $\frac{3}{4}$ (c) $\frac{1}{2}$ (d) $\frac{3}{2}$

ASSERTION & REASON TYPE QUESTIONS:

In the following questions (19 & 20), a statement of Assertion (A) is followed by a statement of Reason(R). Choose the correct answer out of the following choices.

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).

(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(c) (A) is true but (R) is false.

(d) (A) is false but(R) is true.

19. Assertion: The arithmetic mean (A.M.) between two numbers is 34 and their geometric mean is 16. The numbers are 4 and 64.

Reason: For two numbers a and b, A.M. = (a+b)/2 and G.M. = √ab .

20. Assertion: If a and b are non-zero constants, then the derivative of f(x) = ax+b. is a.

Reason: If a, b and c are non-zero constants, then the derivative of f(x) = ax2+bx +c is b.

SECTION – B (2 × 5 =10)

(Very Short Answer type questions of 2 marks each)

21. Solve the following linear inequality: $|3x - 5| \geq 4$.

22. Compute $(98)^5$.

23. Determine x so that 2 is the slope of the line through the points (2, 5) and (x, 3).

24. (a) Evaluate the limit:

$$\lim_{x\to 0}\frac{1-\cos x}{\sin x}$$

OR

(b) Find the derivative of the function f(x) = $(x+\frac{1}{x})^3$.

25. (a) If two letters are chosen at random from the English alphabet , find the probability that both are vowels.

OR

(b) What is the probability of getting a sum of 6 in a single toss of a pair of fair dice ?

SECTION – C (3x6=18)

(Short Answer type questions of 3 marks each)

26. A and B are two sets such that n (A -B) = 14 + x, n (B -A) = 3x and n (A$\cap B) = x$. If n(A) = n(B), find the value of x.

27. Let R = $\{(x, y): x , y \in Z, y = 2x - 4\}$. If (a, -2) and (4, b^2) belong to R, find the values of a and b.

28. (a) Find the domain and range of the function f(x) = $\sqrt{4-x^2}$

OR

(b) Find the domain of the function f(x) = $\frac{1}{\log{(1-x)}} + \sqrt{x+3}$.

29. Write the conjugate of (2+3i) (1-2i) in the form of a+ ib, a, b$\in R$.

30. How many lawn – tennis mixed doubles games can be arranged from 7 married couples if no husband and wife pair is included in the same game?

31. (a) Find the derivative of the function f(x) = sin x from the first principle.

OR

(b) If f(x) = $\frac{x\sin x}{1+\cos x}$, find f'($\frac{\pi}{2}$).

SECTION – D (5x4=20)

(Long Answer type questions of 5 marks each)

32. (a) Prove that : $(1 + \cos\frac{\pi}{8})(1 + \cos\frac{3\pi}{8})\left(1 + \cos\frac{5\pi}{8}\right)\left(1 + \cos\frac{7\pi}{8}\right) = \frac{1}{8}$

OR

(b) If tan x = $-\frac{4}{3}$, x lies in quadrant III, Find the values of $\sin\frac{x}{2}$, $\cos\frac{x}{2}$ and $\tan\frac{x}{2}$.

33. 150 workers were engaged to finish a job in a certain number of days. 4 workers dropped out on second day, 4 more workers dropped out on the third day and so on. It took 8 days more to finish the work. Find the number of days in which the work was completed?

34.The mean and variance of 7 observations are 8 and 16 respectively. If five of observations are 2,4,10,12 and 14, find the remaining two observations.

35. (a) Find the equations of the lines through the point of intersection of the lines

x – y +1 = 0 and 2x – 3y +5 = 0 and whose distance from the point (3,2) is 7/5 units.

OR

(b) Find the equation of the line that has y−intercept 4 and is perpendicular to the line y=3x−2.

SECTION – E (4x3=12)

(Case Study Based questions of 4 marks each with sub-parts)

36. A dentist conducts a team to take surveys of people in his locality about using toothpaste. A survey team has some persons and the survey team owner makes a team out of total persons available at that time. If he has a group of 9 persons available at that time out of which 5 are men and 4 are women.

(i) In the committee, if it is required to seat 5 men and 4 women in a row so that the women occupy the even places. How many such arrangements are possible? (1)

(ii) If P(2n-1, n): P(2n + 1, n − 1) = 22:7, find n. (2)

(iii) From a team of 6 students, in how many ways can we choose a captain and vice- captain assuming one person can not hold more than one position? (1)

37. One of the four persons John, Rita, Aslam or Gurpreet will be promoted next month. Consequently, the sample space consists of four elementary outcomes S = {John promoted, Rita promoted, Aslam promoted, Gurpreet promoted}. You are told that the chances of John's promotion is same as that of Gurpreet, Rita's chances of promotion are twice as likely as Johns. Aslam's chances are four times that of John.

(i) What is the probability that John got promotion? (1)

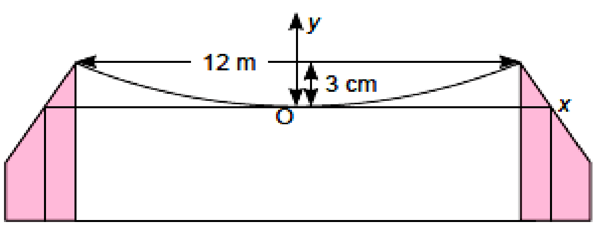
(ii) What is the probability that Rita got promotion? (1)

(iii) (a) What is the probability that Aslam got promotion? (2)

OR

(b) What is the probability that Gurpreet got promotion?

38. A beam is supported at its ends by supports which are 12 m apart. Since the load is concentrated at its centre, there is a deflection of 3 cm at the centre and defected beam is in the shape of parabola. Now considering the centre of beam is at origin as shown in figure. Answer the following:



(i) Write the form of the equation of parabola. (1)

(ii) Find the focus of parabola. (1)

(iii) (a) Find the length of latus rectum of parabola. (2)

OR

(b) How far from the centre is the deflection 1 cm?

SOLUTIONS OF PRACTICE QUESTION PAPER-2

SUBJECT: MATHEMATICS

CLASS - XI

SECTION – A (MCQs) (1×20=20)

Ans. (1): (c) The collection of most talented writers of India.

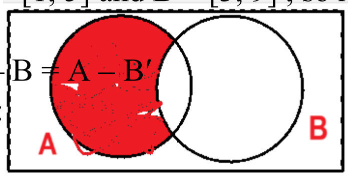
Solution: The collection of most talented writers of India is not well defined.

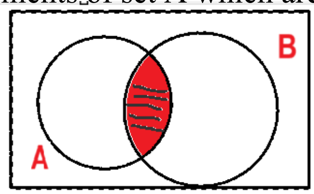
Ans, (2): (d) [1, 3)

Here, A = [1, 5] and B = [3, 9] , so A – B = elements of set A which are not in set B = [1, 3)

Ans. (3): (c) A – B = A – B′

Solution:





A-B $\neq$ A- B'

Ans. (4): (c) $2^{mn} – 1$

Solution: No. of relation from set A to set B = $2^{n(A)\times n(B)}$

Ans. (5): (c) R = {(2,1), (4,3), (6, 5), (8, 7), (10,9)}

Solution: Since every element of set A has unique image in set B for the function f : A→ B.

Ans. (6): (c) Domain= [1, ∞), Range = [0, ∞)

Solution: Function f(x) = $\sqrt{x-1}$ will be real if x – 1 ≥ 0, so x ≥ 1, Domain = [1, ∞)

For range, we have, x – 1 ≥ 0, $\sqrt{x-1} \geq 0$, y ≥ 0, Range = [0, ∞)

Ans (7): (a) $\frac{\pi}{4}$

Solution: Angle turned in 450 revolutions in one hour =450× 2π

So, Angle turned in 60 minutes =900π

Angle turned in 1 minutes = $\frac{900\pi}{60}$

Angle turned in 60 seconds = $\frac{900\pi}{60}$

Angle turned in 1 second = $\frac{900\pi}{60\times60} = \frac{\pi}{4}$ radian

Ans (8): (b) 1

Solution: We have, (tan 1°.tan 2°.tan 3°… tan 44°).tan 45°.(tan 46°……tan 89°)

= (tan (90-89)°.tan (90-88)°…. tan (90-46)°).tan 45°.(tan 46°……tan 89°)

= (cot 89°.cot 88°.cot 87°… cot 49°).1.(tan 46°……tan 89°)

= (cot 89° tan 89°). (cot 88° tan 88°)………..(tan 46°cot 46°)

= 1.1……………….1 = 1

Ans (9): (c) sec θ = $\frac{1}{2}$

Solution: {since range of cos θ is [-1 , 1] }

So the range of sec θ = R - [-1 , 1]

Ans. (10): (a) i

Solution:We have , $\frac{i^{4n+1}-i^{4n-1}}{2} = \frac{(i^4)^n,i-(i^4)^n,i^{-i}}{2} = \frac{1)^n,i-(1)^n,i^{-i}}{2} = \frac{i^2-1}{2i} = \frac{-1-1}{2i} = \frac{-2}{2i} = \frac{-i}{i^2} = \frac{-i}{-1}$ = I

Ans (11): (a) (-2, 0]

Solution: $10 \le -5(x-2) < 20$

$=10 \le -5x + 10 < 20$

$=10 - 10 \le -5x < 20 - 10$

$= 0 \le -5x < 10$

$= \frac{0}{-5} \ge \frac{-5x}{-5} > \frac{10}{-5}$

$= 0 \ge x > -2$

So , x ∈(-2, 0]

Ans (12): (d) breadth $\ge 20\ cm$

Solution: Let breadth = x cm, then length = 2x cm

Perimeter ≥ 120

$2(2x + x) \ge 120$

$2(3x) \ge 120$

$6x \ge 120$

$x \ge 20$, hence breadth $\ge 20\ cm$

Ans (13): (d) $5! \times {}^6P_3$

Solution: no. of ways of sitting 5 boys on alternate places = 5!

Now, we have 6 places for girls to sit so that no two girls can sit together.

Therefore, number of ways of sitting 3 girls out of 6 places = ${}^6P_3$

So, required no of ways = $5! \times {}^6P_3$

Ans (14): (b) 3

Solution: Given series

$9^{1/3} \times 9^{1/9} \times 9^{1/27} \times \ldots\ldots\ldots\ldots\text{to } \infty$

$= 9^{\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\cdots\ldots\ldots to\ \infty} = 9^{\frac{\frac{1}{3}}{1-\frac{1}{3}}} = 9^{\frac{\frac{1}{3}}{\frac{2}{3}}} = 9^{\frac{1}{2}} = 3$

Ans (15): (a) 3 units

Solution: Given point P(3,4,5) .

Draw perpendicular PQ from P on yz-plane , then Q (0, 4, 5)

So, PQ = $\sqrt{(0-3)^2 + (4-4)^2 + (5-5)^2}$ = 3 units

Ans (16): (a) 1

Solution : $\lim_{x \to \frac{3}{2}} [x] = [\frac{3}{2}]$, by taking limit

= 1

Ans (17): (d) 1

Solution : $\lim_{x \to 2} \frac{\log(x-1)}{x-2} = \lim_{h \to 0} \frac{\log(2+h-1)}{h}$ [let x-2 =h & as x→ $2\ then\ h \to 0$]

$$= \lim_{h \to 0} \frac{\log(1+h)}{h} = 1$$

Ans (18): (b) $\frac{3}{4}$

Solution : Sample space (S) = {HH, HT, TH, TT]

A = getting atleast one tail= {HT, TH, TT]

Here, n(S) = 4 , n(A) = 3, Therefore P(A) = $\frac{3}{4}$

Ans (19): (a) Both (A) and (R) are true and (R) is the correct explanation of (A).

Solution: Here , Assertion: The arithmetic mean (A.M.) between two numbers is 34 and their geometric mean is 16. The numbers are 4 and 64, is correct.And Reason: For two numbers a and b, A.M. = $\frac{a+b}{2}$ and G.M. = $\sqrt{ab}$, is also correct and it is the correct explanation of given assertion.

So, correct option is (a).

Ans (20): (c) (A) is true but (R) is false.

Solution: Here, Assertion: If a and b are non-zero constants, then the derivative of f(x)=a x + b is a, is correct.

And Reason: If a, b and c are non-zero constants, then the derivative of f(x) = ax^2+bx +c is b, is false. So, correct option is (c).

SECTION – B (2 marks each)

Ans (21): $3x-5 \geq 4$ or $3x-5 \leq -4$

$\Rightarrow$ $3x \geq 9$ or $3x \leq 1$

$\Rightarrow$ $x \geq 3$ or $x \leq \frac{1}{3}$

so, the solution to the inequality is:

$x \leq \frac{1}{3}$ or $x \geq 3$

$\Rightarrow$ $(-\infty, \frac{1}{3}] \cup [3, \infty)$

Ans (22):

We express 98 as the sum or difference of two numbers whose powers are easier to calculate, and then use Binomial Theorem.

Write 98 = 100 – 2 Therefore,

$(98)^5 = (100 - 2)^5$

$= {}^5C_0 (100)^5 - {}^5C_1 (100)^4 .2 + {}^5C_2 (100)^3 2^2 - {}^5C_3 (100)^2 (2)^3 + {}^5C_4 (100) (2)^4 - {}^5C_5 (2)^5$

$= 10000000000 - 5 \times 100000000 \times 2 + 10 \times 1000000 \times 4 - 10 \times 10000 \times 8 + 5 \times 100 \times 16 - 32$

$= 10040008000 - 1000800032$

$= 9039207968$

Ans (23): Given the points (2,5) and (x,3), and knowing the slope is 2:

$2 = \frac{3-5}{x-2}$

⇒ $2(x-2) = -2$

⇒ $2x=2$

⇒ $X = 1$

Ans (24): (**a**) $\lim_{x\to 0}\frac{1-\cos x}{\sin x} = \lim_{x\to 0}\frac{2sin^2\frac{x}{2}}{2sin\frac{x}{2}\,cos\frac{x}{2}} = \lim_{x\to 0}\tan\frac{x}{2} = \tan\frac{0}{2} = \tan 0^0 = 0$

OR

(**b**) Here, f(x) = $(x+\frac{1}{x})^3$.

$\frac{dy}{dx} = \frac{d}{dx}(x+\frac{1}{x})^3 = 3\ (x+\frac{1}{x})^2 . \frac{d}{dx}(x+\ \frac{1}{x}) = 3\ (x+\frac{1}{x})^2 .(1 - \frac{1}{x^2}).$

Ans (25): (**a**) Probability = $\frac{{}^5C_2}{{}^{26}C_2} = \frac{10}{325} = \frac{2}{65}$

OR

(b) Favourable outcomes: (1,5), (2,4), (3,3), (4,2), (5,1)

⇒ no. of favourable outcomes = 5

Total no. of outcomes = 36

⇒ Probability = $\frac{5}{36}$

SECTION – C (3 marks each)

Ans (26): Given: n(A - B) = 14 + x, n(B - A) = 3x, and n(A) = n(B) & n(A ∩ B) = x

Then , n(A) = n(A - B) + n(A ∩ B) = 14 + x + x = 14 + 2x

& n(B) = n(B - A) + n(A ∩ B) = 3x + x = 4x

Since, n(A) = n(B):

14 + 2x = 4x

⇒ 2x = 14 ⇒ x = 7

Ans (27): Here, R = $\{(x, y): x\ , y\ \in Z, y = 2x - 4\}$. If (a, -2) and (4, b^2)

Given (a, −2) ∈ R ⇒ −2 = 2a - 4 ⇒ 2a = 2 ⇒ a = 1

Given (4, b^2) ∈ R ⇒ $b^2 = 2 \times 4 - 4$ ⇒ $b^2 = 4$ ⇒ b = ±2

Ans (28): (a) Find the domain and range of the function f(x) = $\sqrt{4-x^2}$

For domain of f(x), f(x) must be real number ⇒ $4 - x^2 \geq 0$ ……………………(i)

⇒ $4 \geq x^2$

⇒ $x^2 \leq 4$

⇒ $-2 \leq x \leq 2$

So, D_f = [-2, 2]

For the range of f, let f(x) = y, then y = $\sqrt{4-x^2}$

From (i), we have $4 - x^2 \geq 0$ ⇒ $\sqrt{4-x^2} \geq 0$

⇒ $y \geq 0$

So, range of f = [0, ∞)

OR

(b) Given function f(x) = $\frac{1}{\log(1-x)} + \sqrt{x+3}$.

For the D_f,

log (1-x) is real when (1 -x) > 0 $\Rightarrow 1 > x$ $\Rightarrow x < 1$ …………..(i)

& $\sqrt{x+3}$ is real when x + 3 ≥ 0 ⇒ x ≥ - 3 ……………(ii)

Therefore, from (i) and (ii), D_f = [-3, 1)

Ans (29): Let Z = (2+3i) (1-2i)= (2- 4i +3i – 6i^2)= 2- i – 6 (-1) = 2 – i+6 = 8 -i

So, conjugate of Z = 8 +i

Ans (30): No. of ways of selecting 2 men from 7 men = $^7C_2$

Since same pair are not included in the same game, so no of ways of selecting 2 women from remaining 5 women = $^5C_2$

Therefore , total no. of possible mixed doubles games = $^7C_2 \times {}^5C_2 = \frac{7!}{5!\times 2!} \times \frac{5!}{3!\times 2!} = \frac{7.6.5.4.3!}{2.3!.2} = 210$

Ans (31): (**a**) Given function f(x) = sinx

f(x+h) = sin(x+h)

by the the principle of first derivative , f'(x) = $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$$= \lim_{h\to 0} \frac{\sin(x+h)-\sin(x)}{h}$$

$$= \lim_{h\to 0} \frac{2\cos\left(\frac{x+h+x}{2}\right)\sin\left(\frac{x+h-x}{2}\right)}{h}$$

$$= \lim_{h\to 0} \frac{2\cos\left(\frac{2x+h}{2}\right)\sin\left(\frac{h}{2}\right)}{h}$$

$$= \lim_{\frac{h}{2}\to 0} \left(\frac{\sin\left(\frac{h}{2}\right)}{\frac{h}{2}}\right) \cos\left(\frac{2x+h}{2}\right)$$

[as h→ 0 $then$ $\frac{h}{2}$ →0]

= 1. Cos x

= cos x

OR

(**b**) Given function f(x) = $\frac{x\sin x}{1+\cos x}$ find f'($\frac{\pi}{2}$).

By differentiating w.r.to x,

$$f'(x) = \frac{d}{dx}\left(\frac{x\sin x}{1+\cos x}\right)$$

$$= \frac{(1+\cos x)\frac{d}{dx}(x\sin x)-(x\sin x)\frac{d}{dx}(1+\cos x)}{(1+\cos x)^2}$$

$$= \frac{(1+\cos x)(x\cos x+\sin x)-(x\sin x)(0-\sin x)}{(1+\cos x)^2}$$

$$= \frac{(1+\cos x)(x\cos x+\sin x)+(x\sin^2 x)}{(1+\cos x)^2}$$

So, $f'(\frac{\pi}{2}) = \frac{(1+\cos\frac{\pi}{2})(\frac{\pi}{2}\cos\frac{\pi}{2}+\sin\frac{\pi}{2})+(\frac{\pi}{2}sin^2\frac{\pi}{2})}{(1+\cos\frac{\pi}{2})^2}$

$= \frac{(1+0).1+(\frac{\pi}{2}.1)}{(1+0)^2} = \frac{\pi}{2}$

SECTION – D (5 marks each)

Ans (32): (a) L.H.S : $\left(1+cos\frac{\pi}{8}\right)\left(1+cos\frac{3\pi}{8}\right)\left(1+cos\frac{5\pi}{8}\right)\left(1+cos\frac{7\pi}{8}\right)$

$=\left(1+cos\frac{\pi}{8}\right)\left(1+cos\frac{3\pi}{8}\right)\left(1-cos\frac{3\pi}{8}\right)\left(1-cos\frac{\pi}{8}\right)$

$=\left(1-cos^2\frac{\pi}{8}\right)\left(1-cos^2\frac{3\pi}{8}\right)$

$= sin^2\frac{\pi}{8}\ sin^2\frac{3\pi}{8}$

$= \frac{1}{4}\left(1-cos\frac{\pi}{4}\right)(1-cos\frac{3\pi}{4})$

$=\frac{1}{4}\ (1-\frac{1}{\sqrt{2}})\ (1+\frac{1}{\sqrt{2}})$

$=\frac{1}{4}\left(1-\frac{1}{2}\right) = \frac{1}{4}\times\frac{1}{2} = \frac{1}{8}$ = RHS

OR

(b) we have, $\tan x = -\frac{4}{3}$, $\pi < x < \frac{3\pi}{2}$

$\Rightarrow \frac{\pi}{2} < \frac{x}{2} < \frac{3\pi}{4} \Rightarrow \frac{x}{2}$ lies in quadrant II

Now, $\cos x = \frac{-1}{\sqrt{1+\tan^2 x}} = \frac{-1}{\sqrt{1+\frac{16}{9}}}$ = - 3/5

$\sin\frac{x}{2} = \sqrt{\frac{1-\cos x}{2}} = \sqrt{\frac{1+\frac{3}{5}}{2}} = \frac{2}{\sqrt{5}}$

$\cos\frac{x}{2} = -\sqrt{\frac{1+\cos x}{2}} = -\sqrt{\frac{1-3/5}{2}} = \frac{-1}{\sqrt{5}}$

$\Rightarrow \tan\frac{x}{2} = -2$

Ans (33): Let the work be completed in n days.

As 4 workers dropped out on second day, 4 more dropped on third day and so on, so the number of workers on successive day are

150, 146, 142, …………… which form an A.P. with a = 150 and d = -4

∴ the total number of workers, who worked for one day each during n days, is the sum of the A.P. 150, 146, 142, ………. To n terms

$=\frac{n}{2}[2\times 150+(n-1)(-4)]$ = n (152 – 2n).

Had 150 workers worked on each day, the work would have been finished in (n-8) days.

∴ 150 (n-8) = n (152 – 2n)

$\Rightarrow$ $n^2 - n - 600 = 0$

(n- 25) (n + 24) = 0

n = 25, - 24 but n can not be negative.

So, n = 25

Ans (34): Given - Mean = 8, Variance = 16, 7 observations; 5 values are 2, 4, 10, 12, 14.

Let the missing values be x and y

Mean $\Rightarrow (2+4+10+12+14+x+y)/7 = 8$ $\Rightarrow x + y = 14$ …………….(i)

Variance $\Rightarrow (\Sigma x^2)/7 - 64 = 16$ $\Rightarrow \Sigma x^2 = 560$

Known $\Sigma x^2 = 4 + 16 + 100 + 144 + 196 = 460$ $\Rightarrow x^2 + y^2 = 100$

Now, $(x + y)^2 = x^2 + y^2 + 2xy \Rightarrow 196 = 100 + 2xy \Rightarrow x y = 48 \Rightarrow y = \frac{48}{x}$

……………….(ii)

From (i) and (ii), $x + \frac{48}{x} = 14 \Rightarrow x^2 – 14x + 48 = 0 \Rightarrow (x – 8)(x – 6) = 0$

$\Rightarrow x = 6, 8$

Ans: x and y are 6 and 8.

Ans (35): (a) The equation of the family of lines passing through the point of intersection of the given lines is

$$(x – y + 1) + k (2x – 3y +5) = 0$$

$$(1 + 2k) x – (1 + 3k) y + (1 + 5k) = 0 \qquad ……………(i)$$

For the required members of the family, the perpendicular distance from the point (3,2) to (i) is $\frac{7}{5}$ units

$$\frac{|(1 + 2k)\,3 - (1 + 3k)\,2 + (1 + 5k)|}{\sqrt{(1 + 2k)^2+(1 + 3k)^2}} = \frac{7}{5}$$

$$\frac{|(2+ 5k)|}{\sqrt{(1+4k + 4k^2+1+6k+ 9k^2}} = \frac{7}{5}$$

$$49(13k^2+ 10k +2) = 25(5k+ 2)^2$$

$$12k^2 – 10 k – 2 = 0$$

$$6 k^2 – 5 k – 1 = 0$$

$$(k – 1)(6k + 1) = 0$$

$$k= 1, -\frac{1}{6}$$

from (i) , $3x – 4y + 6 = 0$ & $\frac{2}{3}x- \frac{1}{2}y + \frac{1}{6} = 0$

i.e. $3x – 4y + 6 = 0$ & $4x -3y + 1= 0$

OR

(b) Find the equation of the line that has y−intercept 4 and is perpendicular to the line y=3x−2.

The given equation of the is y=3x−2.

Express the given equation as slope-intercept form y= m x +c

where,

Slope(m) = coefficient of x

$m_1=3$

When the lines are perpendicular, Then the product of slope is −1.

$\therefore m_1 \times m_2= - 1$

$3 \times m_2 = -1$

$m_2 = -\frac{1}{3}$

Given, y−intercept of other line is 4.

Therefore, the required equation of the line using the slope-intercept form y=mx+c.

$y = -\frac{1}{3}x + 4.$

SECTION – E (4 marks each)

Ans (36): (i) There are 9 seats, out of which 4 are at even places and rest are at odd places.

Thus, there ae 4 even places.

So, 4 women can be seated in 4 even places in 4! ways.

In rest of the places, five men can be placed in 5! ways.

Hence, required number of ways = 4! x 5! = 24 x 120 = 2880

(ii) We have, $^{(2n-1)}P_n : {}^{(2n+1)}P_{(n-1)} = 22:7$

$7n^2 - 67n - 30 = 0$

n = 10 or n = - 3/7

so, n = 10 (since n can't be negative and fraction)

(iii) From a team of 6 students, two students are to be chosen in such a way that one student will hold only one position. Here, the no. of ways of choosing a captain and vice captain is the permutation of 6 different things taken 2 at a time.

So, $^6P_2 = \frac{6!}{(6-2)!} = 30.$

Ans (37): Let Event are *J* = John promoted, *R* = Rita promoted, *A* = Aslam promoted, *G* = Gurpreet promoted

Given sample space, *S* = {John promoted, Rita promoted, Aslam promoted, Gurpreet promoted}

i.e., *S* = {J, *R*, *A*, *G*)

It is given that, chances of John's promotion is same as that of Gurpreet.

$P(T) = P(G)$

Rita's chances of promotion are twice as likely as John.

$P(R) = 2P(T)$

and Aslam's chances of promotion are four times that of John.

$P(A) = 4P(D)$

(a) Now, $P(C) + P(R) + P(A) + P(C) = 1$

$\Rightarrow P(J) + 2P(J) + 4P(J) + P(J) = 1 \Rightarrow 8P(J) = 1 \Rightarrow P(T) = P(\text{John Promoted}) = 1/8$

(b) $P(\text{Rita promoted}) = P(R)$

$= 2P(J) = 2 \text{ x } 1/8 = 1/4$

(c) P(Aslam promoted)

$= 4P(J) = 4 \times 1/8 = 1/2$

OR

P(Gurpreet promoted)

$= P(G) = P(J) = 1/8$

Ans (38) :(i) Equation of parabola is $x^2 = 4ay$

(ii) Point $(6, \frac{3}{100})$lies on parabola

$\therefore 36 = 4\,a \times \frac{3}{100} \Rightarrow a = 300$ Focus = (0, 300)

(iii) (a) Length of latus rectum = 4a = 4 × 300 = 1200 m.

OR

(b) Where the deflection is 1 cm. Let the coordinates of point be $(k, \frac{3-1}{100}) = (k, \frac{2}{100})$

$x^2 = 4ay \Rightarrow k^2 = 4 \times 300 \times 2/100$

$\Rightarrow k^2 = 24 \Rightarrow k = 2\sqrt{6}$

$\therefore$ At distance of $2\sqrt{6}$ m from centre deflection is 1 cm.

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PRACTICE QUESTION PAPER-3

MATHEMATICS (041). CLASS XI

SECTION-A

(Multiple Choice Questions. Each question carries 1 mark)

1. Let $A = \{x: x = 3n, n \in N\}, B = \{x: x = 5n, n \in N\}$, then $\overline{A \cap B} equals\ to$
   (A) $\{x: x = 15n, n \in N\}$ (B) $\{x: x = 3n\ or\ x = 5n, n \in N\}$
   (C) $N - \{x: x = 15n, n \in N\}$ (D) $N - \{x: x = 3n\ or\ x = 5n, n \in N\}$
2. Let S = $\{x : x$ is a positive multiple of 3 less than 100$\}$ P = $\{x : x$ is a prime number less than 20$\}$. Then n(S) + n(P) is
   (A) 44 (B) 41 (C) 43 (D) 40
3. Let F_1 be the set of all parallelograms, F_2 be the set of all rectangles, F_3 be the set of all rhombuses and F_4 be the set of all squares. Then which of the following is false
   (A) $F_2 \subseteq F_1$ (B) $F_4 = F_2 \cap F_3$ (C) $F_1 = F_2 \cup F_3 \cup F_4$ (D) $F_4 = F_1 \cap F_2 \cap F_3$
4. The domain of the function f given by $f(x) = \frac{x^2 - 3x + 2}{x^2 - x - 6}$
   (A) R $- \{3, -2\}$ (B) R $- \{-3, 2\}$ (C) R $- [3, -2]$ (D) R $-$ $(3, -2)$
5. **Let n (A) = m, and n (B) = n. Then the total number of non-empty relations that can be defined from A to B is**
   (A) m^n (B) $n^m - 1$ (C) mn – 1 (D) $2^{mn} - 1$
6. The value of $\sin \frac{\pi}{12} + cos \frac{\pi}{12}$ is
   (A) 0.5 (B) 1 (C) $\sqrt{\frac{1}{2}}$ (D) $\sqrt{\frac{3}{2}}$
7. The value of tan 1° tan 2° tan 3° ... tan 89° is
   (A) 0 (B) 1 (C) $\frac{1}{2}$ (D) not defined
8. Which of the following is not correct?
   (A) sin θ = – 1/5 (B) cos θ = 1 (C) sec θ = 1/2 (D) tan θ = 20
9. The modulus of the complex number $Z = -1 - \sqrt{3}\ i$ is
   (A) -2 (B) 1 (C) 3 (D) 2
10. If $-3x + 17 < -13$, then
    (A) x ∈ (10, ∞) (B) x ∈ [10, ∞) (C) x ∈ (– ∞, 10] (D) x ∈ [– 10, 10)
11. If $n_{C_{12}} = n_{C_8}$, then n is equal to
    (A) 20 (B) 12 (C) 6 (D) 30
12. Slope of a line which cuts off intercepts of equal lengths on the axes is
    (A) – 1 (B) 0 (C) 2 (D) 3
13. What is the perpendicular distance of the point P (6, 7, 8) from XY-plane?
    (A) 8 (B) 7 (C) 6 (D) None of these
14. $\lim_{x \to 0} \frac{|x|}{x}$ is equal to

(A) 1 (B) –1 (C) 0 (D) does not exists

15. If $f(x) = 1 + x + \frac{x^2}{2} + \frac{x^3}{3} + \cdots \frac{x^{100}}{100}$, then $f'(1)$ is equal to

(A) $\frac{1}{100}$ (B)100 (C) 0 (D) does not exist

16. The variance of the given data 2, 4, 5, 6, 8, 17 is 23.33. Find the variance of the data 4, 8, 10, 12, 16, 34

(A) 23.23 (B) 25.33 (C) 46.66 (D) 93.32

17. In rolling a die let E = getting even number and F = getting odd number then

(A) E and F are mutually exclusive but not exhaustive

(B) E and F are exhaustive but not mutually exclusive

(C) E and F are mutually exclusive as well as exhaustive

(D) None of these

18. A coin is tossed twice. What is the probability that at least one tail occurs?

(A) $\frac{1}{2}$ (B) $\frac{1}{4}$ C) $\frac{3}{4}$ (D) 1

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of Assertion(A) is followed by a statement of Reason(R). Choose the correct answer out of the following choices.

(A) Both A and R are true and R is the correct explanation of A.

(B) Both A and R are true but R is not the correct explanation of A.

(C) A is true but R is false.

(D) A is false but R is true.

19. Assertion(A): Length of the latus rectum of the parabola $x^2 = 13y$ is 13 units

Reason(R): Length of the latus rectum of the parabola $x^2 = 4ay$ is 4a units.

20. **Assertion(A)**: $\lim_{x \to 2} \frac{x^5 - 32}{x - 2} = 160$

Reason(R) : $\lim_{x \to a} \frac{x^n - a^n}{x - a} = n\, a^{n-1}$

SECTION – B

(This section comprises of very short answer type questions (VSA) of 2 mark each)

21. Determine the domain and range of the relation R defined by

$R = \{(x, x + 5) : x \in \{0, 1, 2, 3, 4, 5\}$

OR

Let $A = \{9, 10, 11, 12, 13\}$ and let f: A → N be defined by f(n) = the highest prime factor of n. Find the range of *f*.

22. Find the value of $\tan 15^o$.

23. Solve the inequality: $2\,(2x + 3) - 10 < 6\,(x - 2)$

24. Given that P (3, 2, –4), Q (5, 4, –6) and R (9, 8, –10) are collinear. Find the ratio in which Q divides PR.

25. Find $\lim_{x \to 2} \frac{3x^2 - x - 10}{x^2 - 4}$

OR

Evaluate $\lim_{x\to 0}\frac{\sin ax+bx}{ax+\sin bx}$, $a, b, a+b \neq 0$

SECTION – C

(This section comprises of short answer type questions (SA) of 3 marks each)

26. Write the following intervals in set-builder form:
(a) (– 3, 0) (b) [6 , 12] (c) (6, 12]

27. The Cartesian product A × A has 9 elements among which are found (–1, 0) and (0, 1). Find the set A and the remaining elements of A × A.

OR

Let A = {1, 2, 3, 4, 5…….14}. Define a relation R from A to A by R = {(x, y): 3x – y = 0, where x, y ∈ A}. Write down its domain, codomain and range.

28. Find the real value of θ for which the expression $\frac{1+i\cos\theta}{1-2i\cos\theta}$ is a real number.

29. Find the sum to n terms of the sequence, 8, 88, 888, 8888…

30. Find the equation of a circle with centre $(2, 2)$ and passes through the point $(4, 5)$.

OR

Find the equation of the ellipse whose vertices are (± 13, 0) and foci are $(\pm 5, 0)$.

31. Find the derivative of the following function $f(x) = \frac{\sin x+\cos x}{\sin x-\cos x}$

OR

Find the derivative of $x^5(5\ \sin x - 3\cos x)$.

SECTION – D

(**This section comprises of long answer type questions (LA) of 5 marks each**)

32. If $\tan x = \frac{3}{4}$ and $\pi < x < \frac{3\pi}{2}$ then find the value of $\sin\frac{x}{2}$, $\cos\frac{x}{2}$ and $\tan\frac{x}{2}$.

OR

a) Prove that: $\cot 4x\,(\sin 5x + \sin 3x) = \cot x\,(\sin 5x - \sin 3x)$

b) Prove that: $\tan 3x\ \tan 2x\ \tan x = \tan 3x - \tan 2x - \tan x$

33. (a) Using Binomial Theorem, find which number is greater $(1.1)^{10000}$ or 1000.
(b) Find $(a+b)^4 - (a-b)^4$. Hence evaluate $(\sqrt{3}+\sqrt{2})^4 - (\sqrt{3}-\sqrt{2})^4$.

34. If the Arithmetic Mean of two positive numbers are 3 times their Geometric Mean , prove that they are in the ratio $(3+2\sqrt{2}) : (3-2\sqrt{2})$.

35. Calculate the Mean Deviation about Mean for the following distribution

Marks	0-100	100-200	200-300	300-400	400-500	500-600	600-700	700-800

No of students	4	8	9	10	7	5	4	3

OR

Calculate Mean and Standard deviation for the following

Age	20-30	30-40	40-50	50-60	60-70	70-80	80-90
No of persons	3	6	13	15	14	5	4

SECTION –E

(**This section comprises of 3 case -study/passage-based questions of 4 marks each with two sub-parts. The first and second case study question has three sub-parts of 1, 1 and 2 marks respectively. Third case study questions consist of two sub-parts of 2 marks each**.)

36. Four friends are playing with cards. They are choosing 4 cards from a pack of 52 playing cards. Using this information answer the following questions

(a) How many of these four cards are of the same suit?
(b) How many of these four cards are face cards?
(c) How many of these two are red cards and two are black cards?

OR

How many of these four cards are of the same colour?

37. **Case Study II:** A committee of two persons is selected from two men and two women.

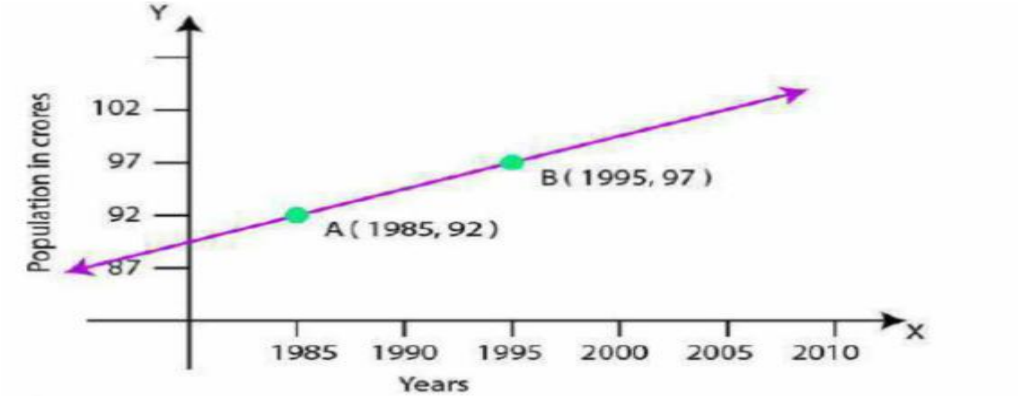
Using this information answer the following questions.

(a) Find the Probability that the committee will have no men.
(b) Find the Probability that the committee will have two men.
(c) Find the Probability that the committee will have one man

OR

Find the Probability that the committee will have atleast one woman

38. **Case Study III**: Consider the following population and year graph and using it,



a) Find equation of line parallel to AB passing through (0, 85)

b) What was the population in the year 2010.

ANSWERS FOR PRACTICE QUESTION PAPER -3
CLASS XI - MATHEMATICS

SECTION-A
(Multiple Choice Questions)

1. (C) $N - \{x: x = 15n, n \in N\}$,
   $A \cap B = 15n$, $(A \cap B)' = N - \{x: x = 15n, n \in N\}$
2. (B) 41; n(S)=33, n(P)=8
3. (C) $F_1 = F_2 \cup F_3 \cup F_4$; Some parallelograms are left out in F_2, F_3, F_4
4. (A) $R - \{3, -2\}$; $x^2 - x - 6 = (x-3)(x+2)$
5. **(D)** $\mathbf{2^{mn} - 1}$: A×B contains m×n elements.
   No. of relations from A to B is $\mathbf{2^{mn}}$. No. of non- empty relations $\mathbf{2^{mn} - 1}$.
6. (C) $sin\frac{\pi}{12} + cos\frac{\pi}{12} = cos\frac{5\pi}{12} + cos\frac{\pi}{12} = 2cos\frac{3\pi}{12}\ cos\frac{4\pi}{12} = \frac{1}{\sqrt{2}}$
7. (B) 1 ; tan 1° tan 2° tan 3° ... tan 89°= tan 1° tan 89° tan 2° tan 88° ……= tan 1° cot 1° tan 2° cot2° ……=1
8. (C) $\sec\theta = \frac{1}{2}; \cos\theta = 2$, not possible
9. (D) 2; $\sqrt{(-1)^2 + (-\sqrt{3})^2} = 2$
10. (A) $x \in (10, \infty)$; 3x > 30 => x > 10
11. (A) 20; $n_{C_{12}} = n_{C_{n-12}} = n_{C_8}, n - 12 = 8, n = 20$
12. (A) – 1; $point\ (a, 0) and (0, a).\ tan\theta = \frac{a-0}{0-a}$=-1
13. (A) 8 , Distance along z-axis is 8
14. (D) does not exists; $\lim_{x\to0+}\frac{|x|}{x} = 1\ but\ \lim_{x\to0-}\frac{|x|}{x} = -1$
15. (B)100; $f'(x)$ consists 100 terms and $f'(1)$ consists 100 one i.e. $100 \times 1 = 100$
16. (D) 93.32; New variance= k^2x Old variance=2^2x 23.33=93.32
17. (C) E and F are mutually exclusive as well as exhaustive; F ={1,3,5}, E={2,4,6}
18. (C) $\frac{3}{4}$; S={HH, HT, TH, TT}, Event(E)= {HT, TH, TT}

ASSERTION-REASON BASED QUESTIONS

19. (A) Both **A** and **R** are true and R is the correct explanation of A.
20. (D) **A** is false but **R** is true; $\lim_{x\to2}\frac{x^5-32}{x-2} = 5 \times 2^{5-1} = 80$.

SECTION – B

21. R = { (0,5), (1,6), (2,7), (3,8), (4,9), (5,10)}
    Domain = { 0,1,2,3,4,5} Range = { 5,6,7,8,9,10 }

    OR

    f = {(9,3), (10,5), (11,11), (12,3), (13,13)} Range = { 3,5,11,13}

22. $\tan(60° - 45°) = \frac{\tan 60° - \tan 45°}{1 + \tan 60° \tan 45°} = \frac{\sqrt{3}-1}{\sqrt{3}+1} = 2 - \sqrt{3}$

23. $4x + 6 - 10 < 6x - 12$
$\Rightarrow -2x < -8$

$\Rightarrow x > 4$

24. Assume that the point Q (5, 4, –6) divides the line segment joining points P (3, 2, –4) and R (9, 8, –10) in the ratio k:1. Therefore, by using the section formula, we get

(5, 4, -6) = $\left(\frac{k(9)+3}{k+1}, \frac{k(8)+2}{k+1}, \frac{k(-10)-4}{k+1}\right) \Rightarrow \frac{9k+3}{k+1} = 5$

$\Rightarrow 9k+3 = 5k+5$

$\Rightarrow 4k = 2 \quad \Rightarrow k = \frac{2}{4} = \frac{1}{2}$

Therefore, the value of k is ½. Hence, the point Q divides PR in the ratio of 1:2.

25. $\lim_{x\to 2} \frac{3x^2-6x+5x-10}{(x+2)(x-2)} = \lim_{x\to 2} \frac{(3x+5)(x-2)}{(x+2)(x-2)} = \frac{11}{4}$

OR

$\lim_{x\to 0} \frac{\sin ax+bx}{ax+\sin bx} = \lim_{x\to 0} \frac{\frac{\sin ax}{ax}\times ax+bx}{ax+\frac{\sin bx}{bx}\times bx} = \lim_{x\to 0} \frac{\frac{\sin ax}{ax}\cdot a+b}{a+\frac{\sin bx}{bx}\cdot b}$

$= \frac{1\cdot a+b}{a+1\cdot b} = \frac{a+b}{a+b} = 1$

SECTION – C

26. (a) $(-3, 0) = \{x : x \in R \text{ and } -3 < x < 0\}$
(b) $[6, 12] = \{x : x \in R \text{ and } 6 \le x \le 12\}$
(c) $(6, 12] = \{x : x \in R \text{ and } 6 < x \le 12\}$

27. For $A = \{-1, 0, 1\}$

$A\times A = \{(-1,-1), (-1,0), (-1,1), (0,-1), (0,0), (0,1), (1,-1), (1,0), (1,1)\}$

Remaining elements $\{(-1,-1), (-1,1), (0,-1), (0,0), (1,-1), (1,0), (1,1)\}$

OR

$R = \{(1,3), (2,6), (3,9), (4,12)\}$

For domain= $\{1,2,3,4\}$, Codomain = $\{1,2,3, \dots \dots, 14\}$ and range= $\{3,6,9,12\}$

28. $z = \frac{1+i\cos\theta}{1-2i\cos\theta} \times \frac{1+2i\cos\theta}{1+2i\cos\theta} = \frac{1+2i\cos\theta+i\cos\theta+2i^2\cos^2\theta}{1-4i^2\cos^2\theta}$

$= \frac{(1-2\cos^2\theta)+i(3\cos\theta)}{1+4\cos^2\theta}$

The given number is a real number hence the imaginary part of the complex number is zero

$\therefore \frac{(3\cos\theta)}{1+4\cos^2\theta} = 0 \quad \Rightarrow \therefore \cos\theta = 0$

$\Rightarrow \therefore \theta = 2k\pi \pm \frac{\pi}{2}; \; k\epsilon Z$

29. $S = 8 + 88 + 888 + 8888 + \cdots ..\text{to n terms}$

$= 8(1 + 11 + 111 + 1111 + \cdots ..\text{to n terms})$

$= \frac{8}{9}(9 + 99 + 999 + 9999 + \cdots \dots \dots \text{to n terms})$

$= \frac{8}{9}[(10-1)+(100-1)+(1000-1)+\cdots \ldots\ldots \text{to n terms})]$

$= \frac{8}{9}[(10+100+1000+\cdots \text{to n terms})-(1+1+1+\cdots \text{to nterms})]$

$= \frac{8}{9}\left[\frac{10^{n+1}-10-9n}{9}\right] = \frac{8}{81}(10^{n+1}-10-9n)$

30. Radius $= \sqrt{(4-2)^2+(5-2)^2} = \sqrt{13}$ units

$(x-2)^2+(y-2)^2 = (\sqrt{13})^2 \Rightarrow x^2+y^2-4x-4y-5=0$

OR

Ellipse is horizontal and in the form of $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

a = 13 and c = 5 ⇒ $c^2 = a^2-b^2 \Rightarrow 25 = 169-b^2 \Rightarrow b = 12$

Equation is $\frac{x^2}{169}+\frac{y^2}{144}=1$

31. By quotient rule

$$f'(x) = \frac{(\sin x-\cos x)\frac{d}{dx}(\sin x+\cos x)-(\sin x+\cos x)\frac{d}{dx}(\sin x-\cos x)}{(\sin x-\cos x)^2}$$

$= \frac{-(\sin x-\cos x)^2-(\sin x+\cos x)^2}{(\sin x-\cos x)^2}$

$= \frac{-2(\sin^2 x+\cos^2 x)}{(\sin x-\cos x)^2}$

$= \frac{-2}{(\sin x-\cos x)^2}$

OR

$4x^2(5\ \sin x-3\cos x)+x^5(5\ \cos x+3\sin x)$

$(20x^2+3x^5)sinx+(5x^5-12x^2cosx)$

SECTION – D

32. $cosx = \frac{-4}{5}\ \ (\pi < x < \frac{3\pi}{2})$

$cosx = 2cos^2\frac{x}{2}-1 => cos^2\frac{x}{2} = \frac{1+cosx}{2} = \frac{1}{10} => cos\frac{x}{2} = \pm\frac{1}{\sqrt{10}}$

$cos\frac{x}{2} = -\frac{1}{\sqrt{10}}\ \ (\frac{\pi}{2} < \frac{x}{2} < \frac{3\pi}{4})$

$\sin\frac{x}{2} = \sqrt{1-\cos^2\frac{x}{2}} = \frac{3}{\sqrt{10}}$

$\tan\frac{x}{2} = -3$

OR

(a) $\cot 4x(\sin 5x + \sin 3x) = \cot 4x(2\sin 4x\cos x) = 2\cos 4x\cos x$

$\cot x(\sin 5x - \sin 3x) = \cot x(2\sin 2x\cos 4x) = 2\cos 4x\cos x$

$\cot 4x(\sin 5x + \sin 3x) = \cot x(\sin 5x - \sin 3x)$

(b) $\tan 3x = \tan(2x + x) = \frac{\tan 2x + \tan x}{1 - \tan 2x\tan x}$

$\tan 3x\tan 2x\tan x = \tan 3x - \tan 2x - \tan x$

33. (a) $(1.1)^{10000} = (1 + 0.1)^{10000} = 1 + 10000X\ 0.1 + \ldots\ldots\ldots + (0.1)^{10000}$

$= 1001 +$ higher order terms .

$(1.1)^{10000}$ is greater than 1000

(b) $(a+b)^4 - (a-b)^4$

$= (a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^3) - (a^4 - 4a^3b + 6a^2b^2 - 4ab^3 + b^3)$

$= 8ab(a^2 + b^2)$

Substituting the values of a and b as $\sqrt{3}$ and $\sqrt{2}$ in above we get

$8\sqrt{3} \times \sqrt{2}(3 + 2) = 40\sqrt{6}$

34. Writing $\frac{a+b}{2} = 3\sqrt{ab}$

$\frac{a+b}{2\sqrt{ab}} = \frac{3}{1} \Rightarrow \frac{(\sqrt{a} + \sqrt{b})^2}{\sqrt{a} + \sqrt{b})^2} = \frac{3+1}{3-1} \Rightarrow 2\frac{\sqrt{a} + \sqrt{b}}{\sqrt{a} - \sqrt{b}} = \frac{\sqrt{2}}{1}$

Applying componendo and dividendo to get $\frac{\sqrt{a}}{\sqrt{b}} = \frac{\sqrt{2}+1}{\sqrt{2}-1}$

Squaring and getting the result as $\frac{a}{b} = \frac{(\sqrt{2}+1)^2}{\sqrt{2}-1)^2} = \frac{3+2\sqrt{2}}{3-2\sqrt{2}}$

class	f_i	x_i	d_i	$f_i d_i$	$\lvert x_i - \bar{x}\rvert$	$f_i\lvert x_i - \bar{x}\rvert$
0-100	4	50	-4	-16	308	1232
100-200	8	150	-3	-24	208	1664
200-300	9	250	-2	-18	108	072
300-400	10	350	-1	-10	8	80
400-500	7	450	0	0	92	644
500-600	5	550	1	5	192	960
600-700	4	650	2	8	292	1168
700-800	3	750	3	9	392	1176
TOTAL	50			-46		7896

35.

Mean=358						
Mean deviation=157.92						

OR

class	f_i	x_i	$u_i = \frac{xi-55}{10}$	f_iu_i	u_i^2	fiu_i^2
20-30	3	25	-3	-9	9	27
30-40	6	35	-2	-12	4	24
40-50	13	45	-1	-13	1	13
50-60	15	55	0	0	0	0
60-70	14	65	1	14	1	14
70-80	5	75	2	10	4	20

80-90	4	85	3	12	9	36
TOTAL	60			2		134
Mean=55.3	Var =223.2	SD = 14.94				

Mean = AM + ΣfdN×**Class size** = **55**+**260**× **10**=**55.3**

Variance=[$\Sigma fd2N$−(ΣfdN)**2**]×$class\ size$**2**=[**13460**−(**260**)**2**]×**100**=**803636**=**223.2**

Standard Deviation=√**223.2**=**14.94**

SECTION –E

36. (a) Required number of ways $= {}^{13}C_4 + {}^{13}C_4 + {}^{13}C_4 + {}^{13}C_4 = 4 \times {}^{13}C_4 = 2860$

(b) Required number of ways $= {}^{12}C_4 = 495$

(c) Required number of ways $= {}^{26}C_2 \times {}^{26}C_2 = 105625$

OR

Required number of ways$= {}^{26}C_4 + {}^{26}C_4 = 2 \times {}^{26}C_4 = 29900$

37. P(no men) = $\frac{2_{C_2}}{4_{C_2}} = \frac{1}{6}$

P(two men) = $\frac{2_{C_2}}{4_{C_2}} = \frac{1}{6}$

P(one man) = $\frac{(2_{C_1})(2_{C_1})}{4_{C_2}} = \frac{2}{3}$

OR

P(at least one woman) = $\frac{(2_{C_1})(2_{C_1})+2_{C_2}}{4_{C_2}} = \frac{5}{6}$

38. (a) Slope of the line AB = $\frac{97-92}{1995-1985} = \frac{5}{10} = \frac{1}{2}$

Equation will be $y - 85 = \frac{1}{2}(x - 0)$ or $x - 2y + 170 = 0$

(b) Let population in 2010 be P, then $\frac{P-97}{2010-1995} = \frac{1}{2}$

$P = 104.5$ crore